ROB 430/599.430: Deep Learning for Robot Perception and Manipulation (DeepRob)

Lecture 16: Sequence Prediction (RNNs, Seq2Seq, etc.); Attention and Transformers

03/16/2026

https://deeprob.org/w26/

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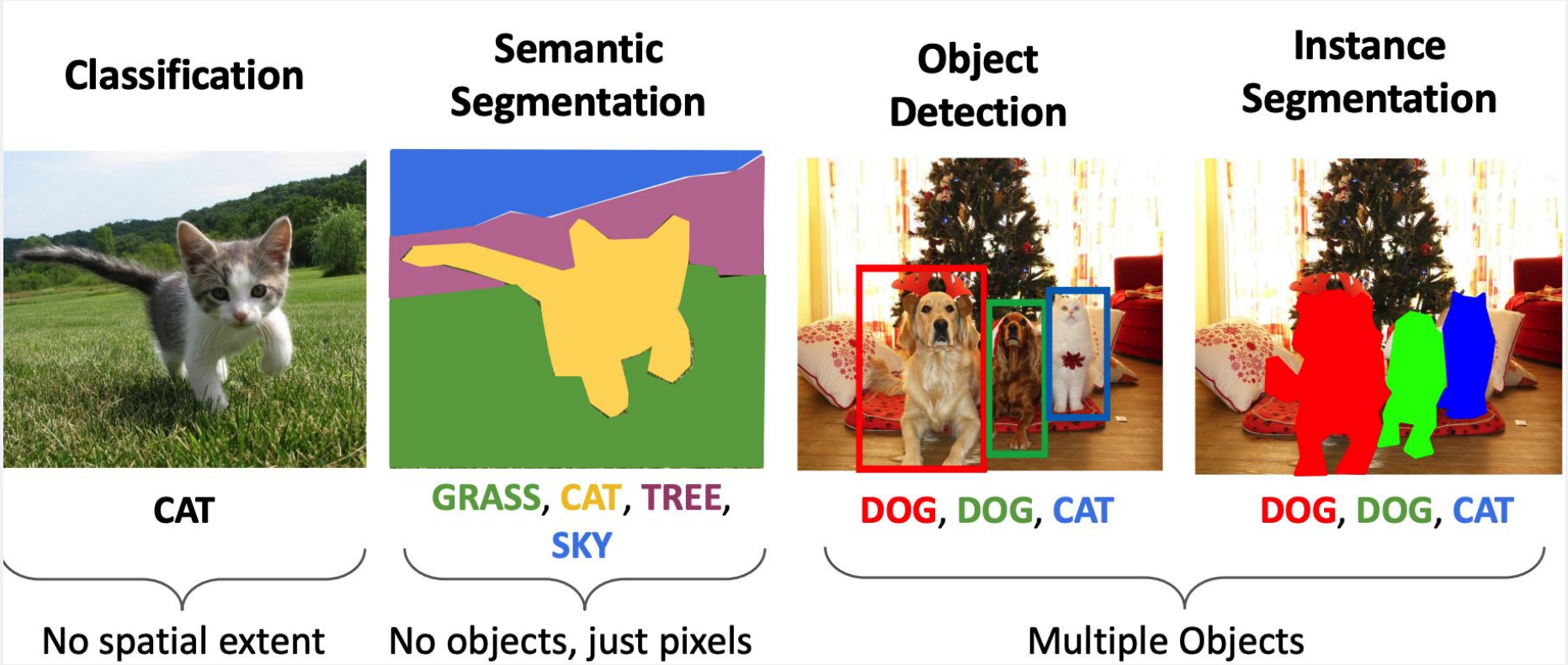
Today

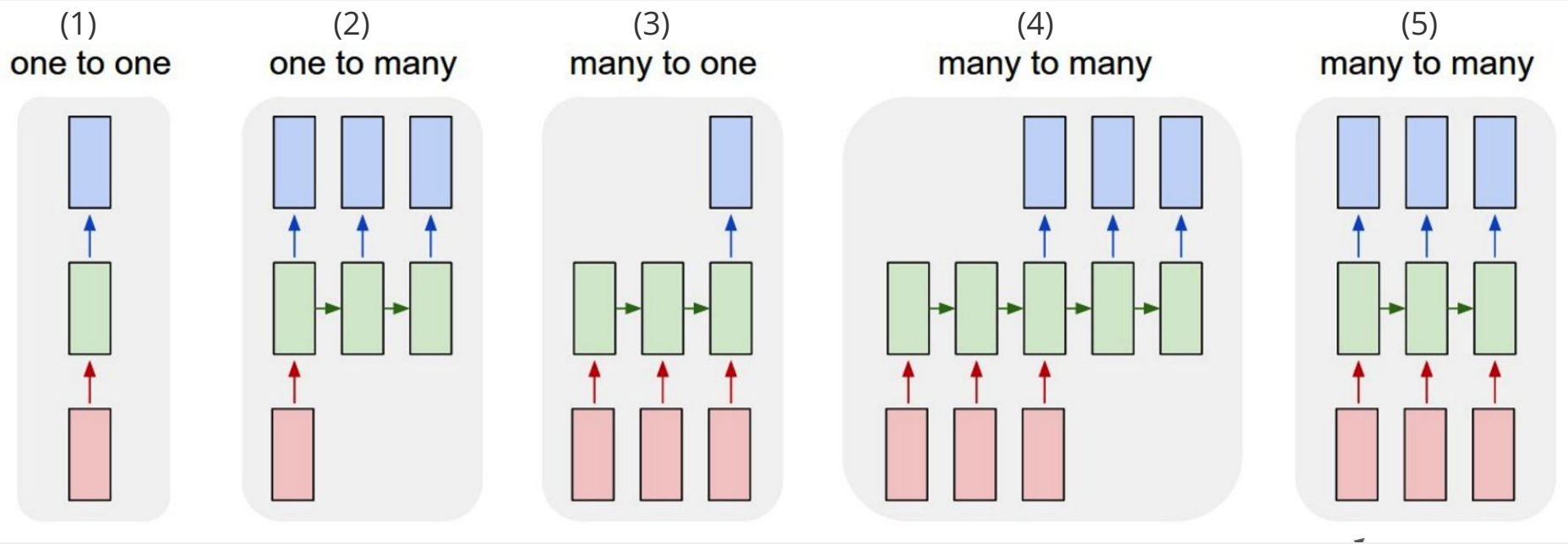
P4 released

- Feedback and Recap (5min)
- Processing Sequences (1hr10min)
  - RNN
  - LSTM
  - Seq2Seq
  - Attention
  - Transformer
  - ViT (if time allows)
- Summary and Takeaways (5min)

So far…


Classification
Semantic Segmentation
Object Detection
Instance Segmentation
CAT
GRASS, CAT, TREE, SKY
DOG, DOG, CAT
DOG, DOG, CAT
No spatial extent
No objects, just pixels
Multiple Objects

Recurrent Neural Networks

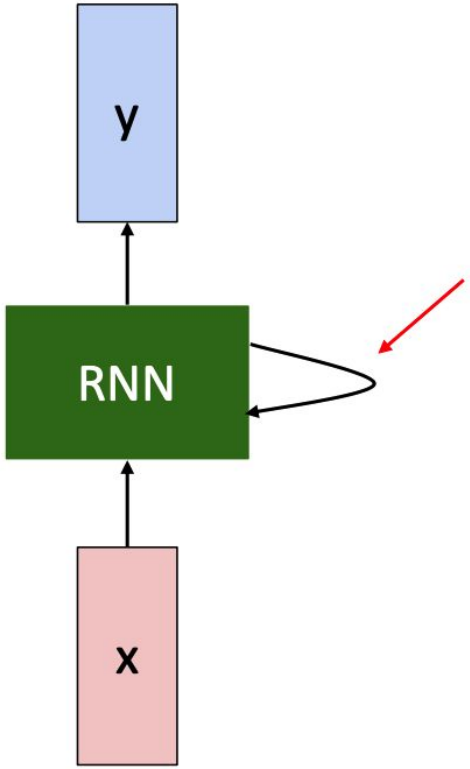


Aha Slides
(In-class participation)

https://ahaslides.com/ATZ7W

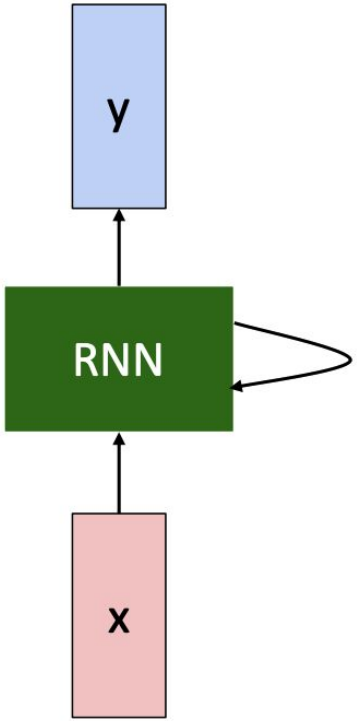
Q1: what are some examples of applications?

Recurrent Neural Networks



Key idea: RNNs have an "internal state" that is updated as a sequence is processed

Recurrent Neural Networks



We can process a sequence of vectors **x** by applying a **recurrence formula** at every time step:

$$h_t = f_W(h_{t-1}, x_t)$$

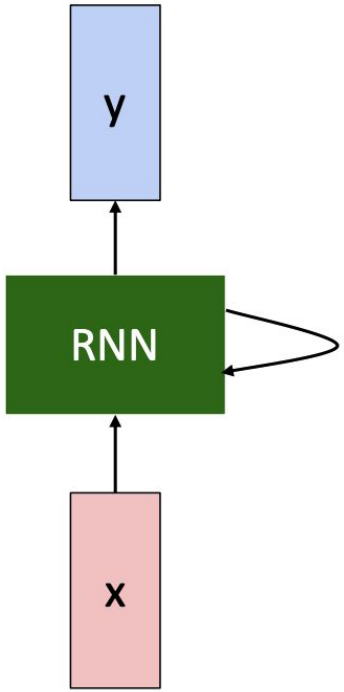
new state

some function with parameters W

old state

input vector at some time step

(Vanilla) Recurrent Neural Networks



The state consists of a single *"hidden"* vector **h**:

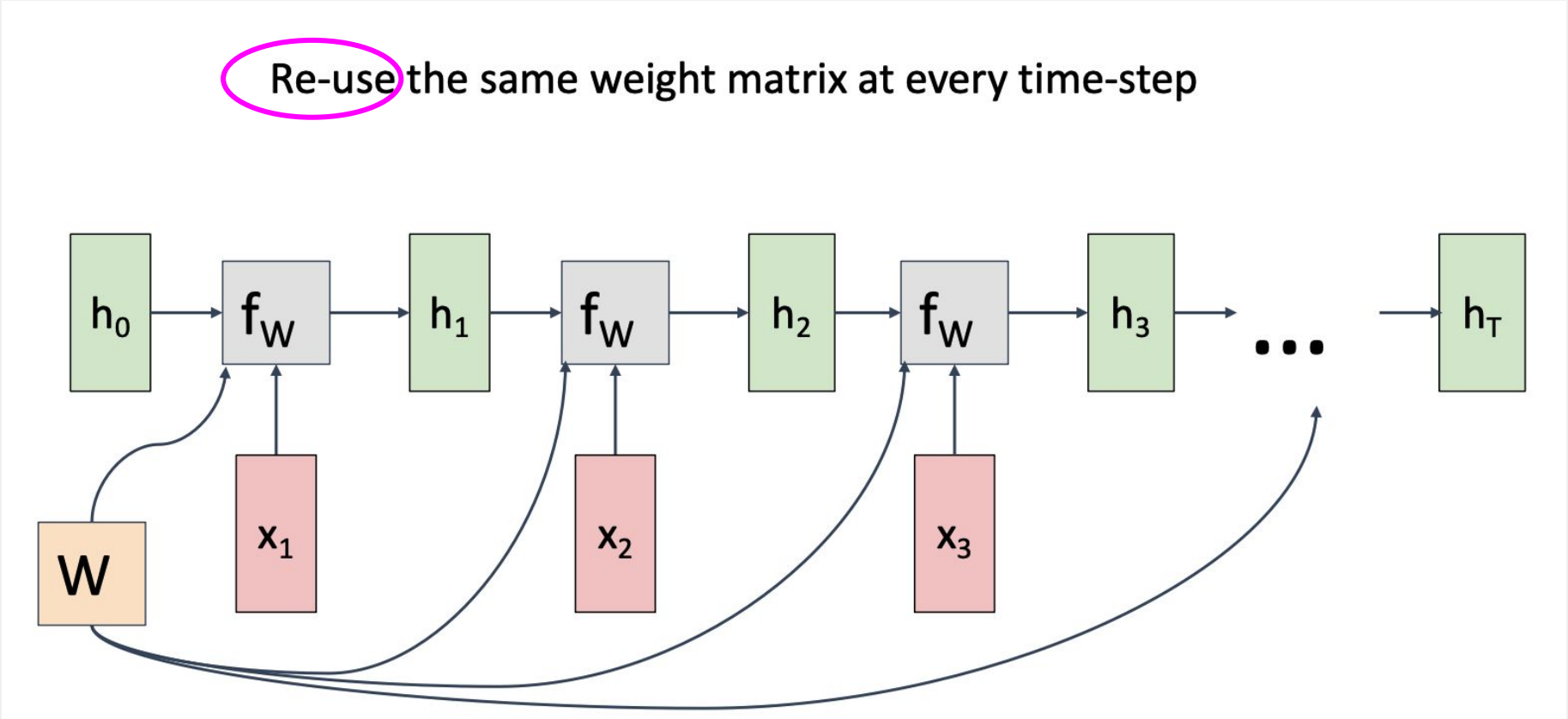
$$h_t = f_W(h_{t-1}, x_t)$$

$$h_t = \tanh(W_{hh}h_{t-1} + W_{xh}x_t + b_h)$$

$$y_t = W_{hy}h_t + b_y$$

Sometimes called a "Vanilla RNN" or an "Elman RNN" after Prof. Jeffrey Elman

Recurrent Neural Networks: computational graph



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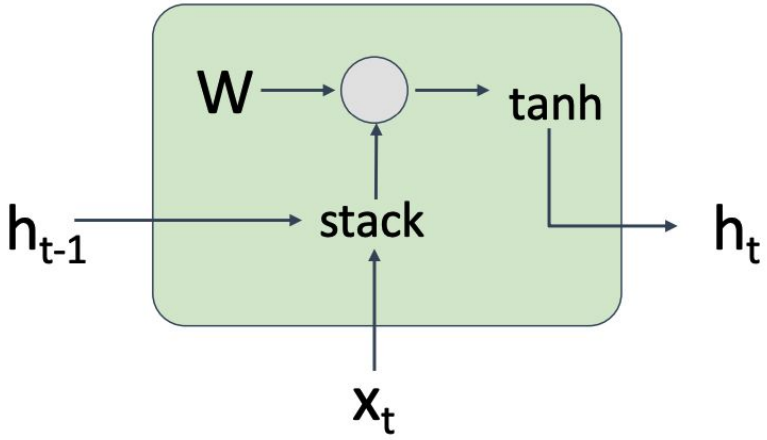
Q2: why re-use (shared) weights?

(Vanilla) Recurrent Neural Networks: Gradient Flow

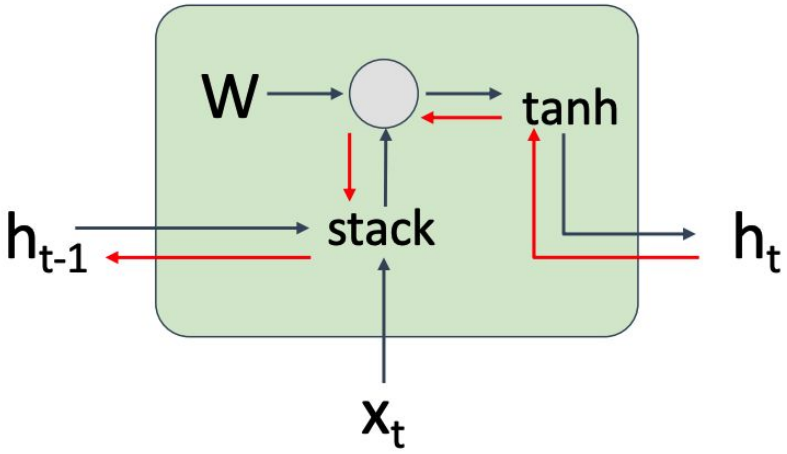


$$h_t = \tanh(W_{hh}h_{t-1} + W_{xh}x_t + b_h)$$
$$= \tanh\left((W_{hh} \quad W_{hx})\begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix} + b_h\right)$$
$$= \tanh\left(W\begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix} + b_h\right)$$

Bengio et al, “Learning long-term dependencies with gradient descent is difficult”, IEEE Transactions on Neural Networks, 1994 https://ieeexplore.ieee.org/document/279181
Pascanu et al, “On the difficulty of training recurrent neural networks”, ICML 2013 https://proceedings.mlr.press/v28/pascanu13.html

(Vanilla) RNN: Gradient Flow

Backpropagation from h_t to h_{t-1} multiplies by W (actually W_{hh}^T)



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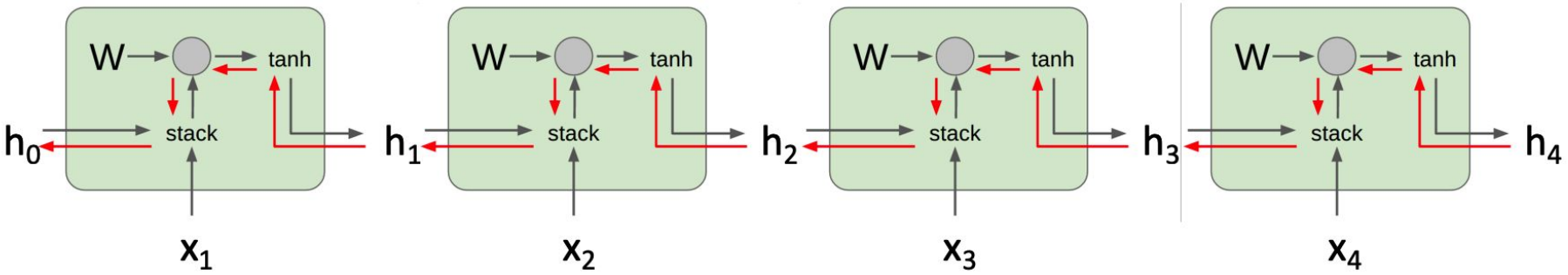
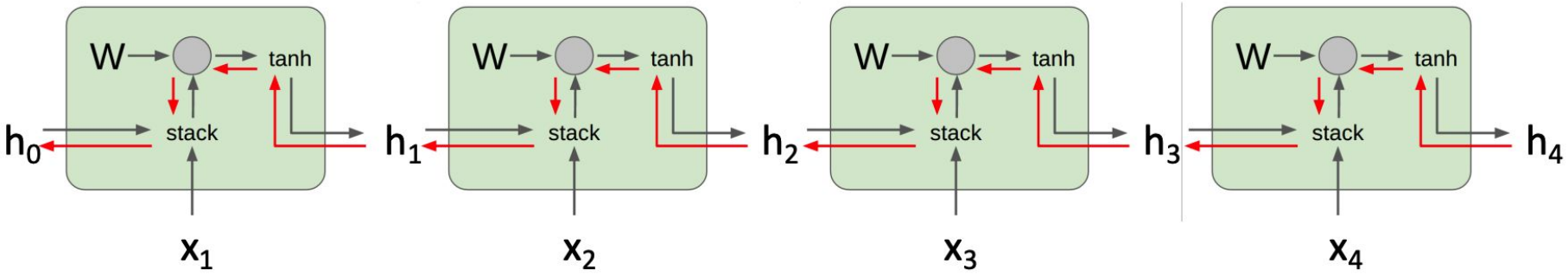


Computing gradient of h_0 involves many factors of W (and repeated tanh)

(Vanilla) RNN: Gradient Flow

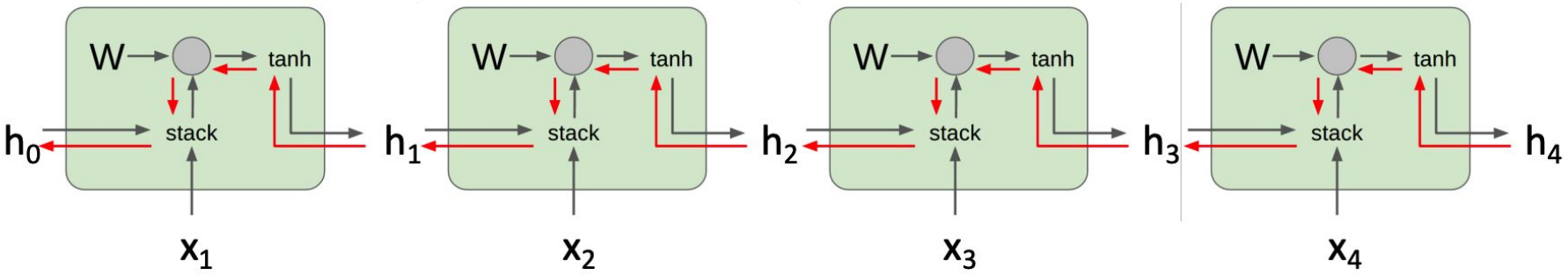


Computing gradient of h_0 involves many factors of W (and repeated tanh)

Largest singular value > 1:
Exploding gradients

Largest singular value < 1:
Vanishing gradients

(Vanilla) RNN: Gradient Flow



Computing gradient of h_0 involves many factors of W (and repeated tanh)

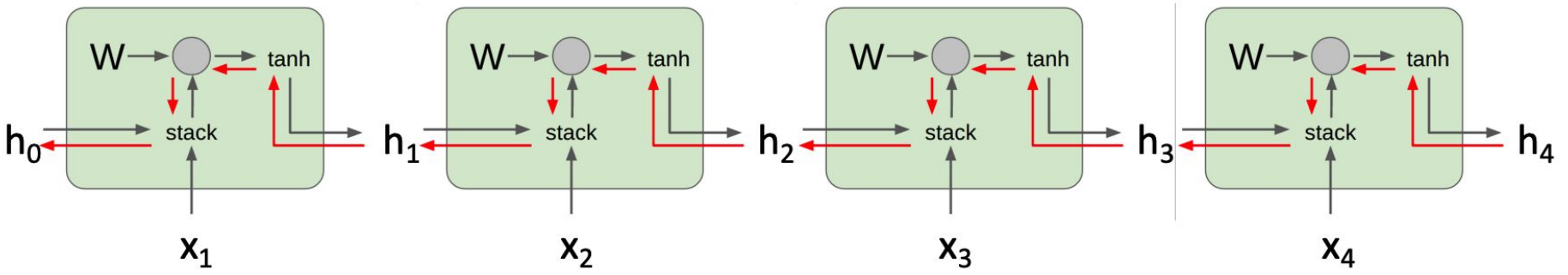
Largest singular value > 1: **Exploding gradients**

→ **Gradient clipping**: Scale gradient if its norm is too big

Largest singular value < 1: **Vanishing gradients**

```
grad_norm = np.sum(grad * grad)
if grad_norm > threshold:
  grad *= (threshold / grad_norm)
```

(Vanilla) RNN: Gradient Flow



Computing gradient of h_0 involves many factors of W (and repeated tanh)

Largest singular value > 1:
Exploding gradients

Largest singular value < 1:
Vanishing gradients → **Change RNN architecture!**

Long Short Term Memory (LSTM)

Long-range dependency

Vanilla RNN

$$h_t = \tanh\left(W \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix} + b_h\right)$$

LSTM

$$\begin{pmatrix} i_t \\ f_t \\ o_t \\ g_t \end{pmatrix} = \begin{pmatrix} \sigma \\ \sigma \\ \sigma \\ \tanh \end{pmatrix} \left(W \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix} + b_h\right)$$

$$c_t = f_t \odot c_{t-1} + i_t \odot g_t$$

$$h_t = o_t \odot \tanh(c_t)$$

Two vectors at each timestep:

Cell state: $c_t \in \mathbb{R}^H$

Hidden state: $h_t \in \mathbb{R}^H$

Hochreiter and Schmidhuber, "Long Short Term Memory", Neural Computation 1997 https://www.bioinf.jku.at/publications/older/2604.pdf

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Long Short Term Memory (LSTM)

Compute four "gates" per timestep:

Input gate: $i_t \in \mathbb{R}^H$

Forget gate: $f_t \in \mathbb{R}^H$

Output gate: $o_t \in \mathbb{R}^H$

"Gate?" gate: $g_t \in \mathbb{R}^H$

LSTM

$$\begin{pmatrix} i_t \\ f_t \\ o_t \\ g_t \end{pmatrix} = \begin{pmatrix} \sigma \\ \sigma \\ \sigma \\ \tanh \end{pmatrix} \left(W \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix} + b_h \right)$$

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Hochreiter and Schmidhuber, "Long Short Term Memory", Neural Computation 1997 https://www.bioinf.jku.at/publications/older/2604.pdf

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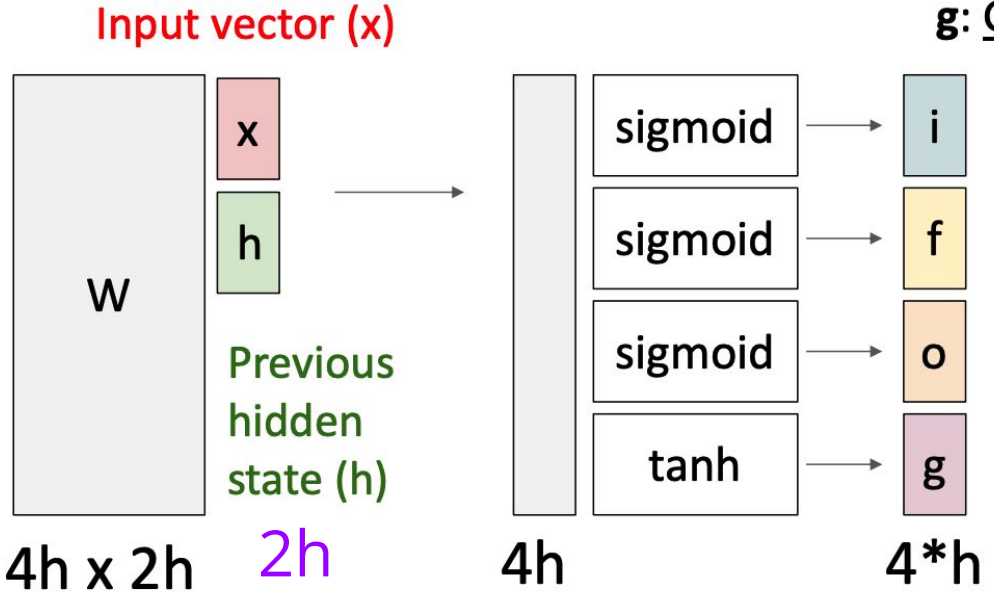
Long Short Term Memory (LSTM)

i: Input gate, whether to write to cell
f: Forget gate, Whether to erase cell
o: Output gate, How much to reveal cell
g: Gate gate (?), How much to write to cell



$$\begin{pmatrix} i_t \\ f_t \\ o_t \\ g_t \end{pmatrix} = \begin{pmatrix} \sigma \\ \sigma \\ \sigma \\ \tanh \end{pmatrix} \left(W \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix} + b_h \right)$$

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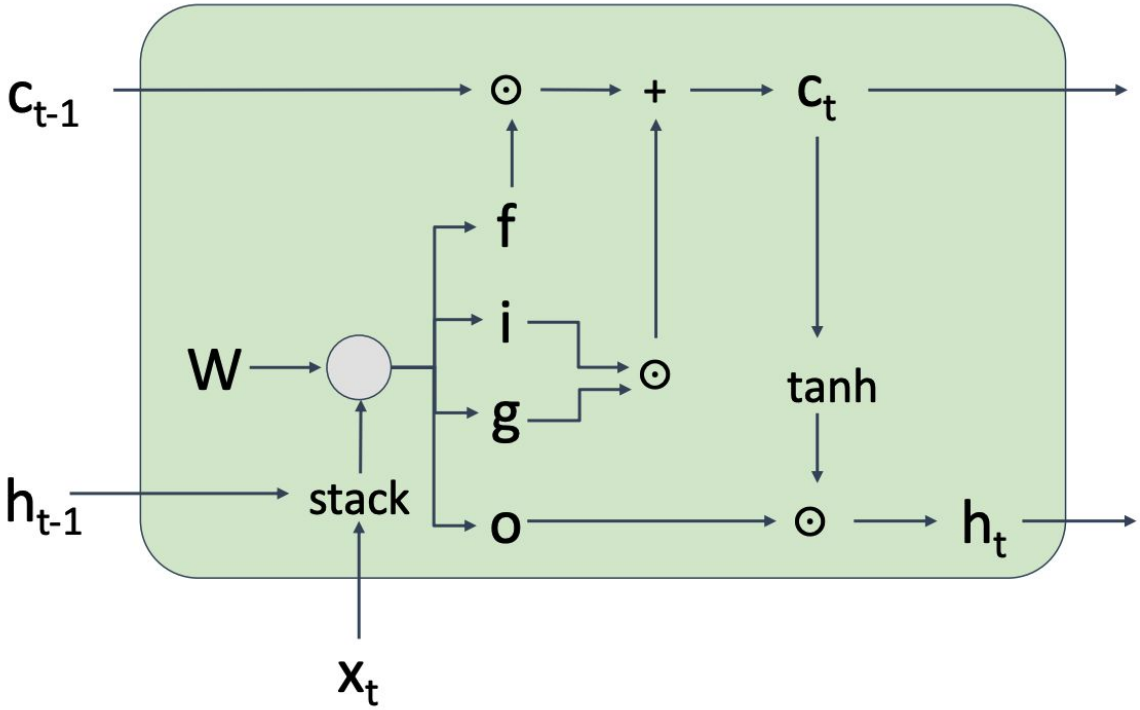
Long Short Term Memory (LSTM)



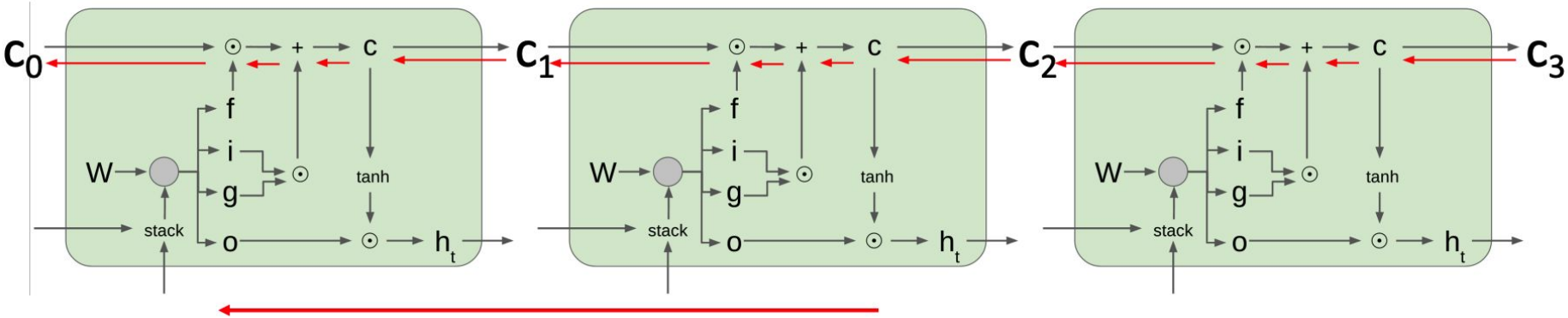
$$\begin{pmatrix} i_t \\ f_t \\ o_t \\ g_t \end{pmatrix} = \begin{pmatrix} \sigma \\ \sigma \\ \sigma \\ \tanh \end{pmatrix} \left(W \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix} + b_h \right)$$

$$c_t = f_t \odot c_{t-1} + i_t \odot g_t$$

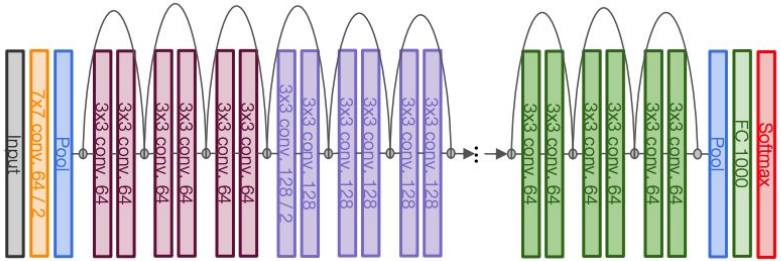
$$h_t = o_t \odot \tanh(c_t)$$

Long Short Term Memory (LSTM)

Uninterrupted gradient flow!



Similar to ResNet!



"Highway network"

https://static.googleusercontent.com/media/research.google.com/en//pubs/archive/46171.pdf

https://arxiv.org/pdf/1505.00387

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Single-Layer RNN

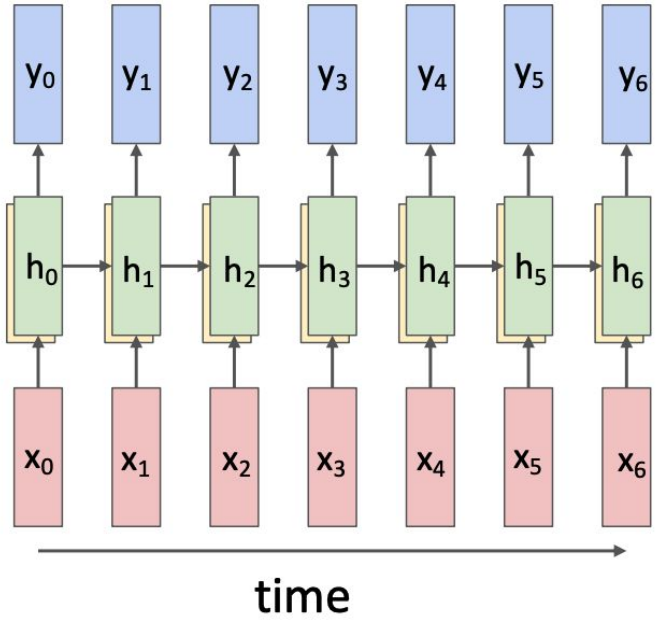
$$h_t = \tanh\left(W\begin{pmatrix}h_{t-1}\\ x_t\end{pmatrix} + b_h\right)$$

LSTM:

$$\begin{pmatrix}i_t\\ f_t\\ o_t\\ g_t\end{pmatrix} = \begin{pmatrix}\sigma\\ \sigma\\ \sigma\\ \tanh\end{pmatrix}\left(W\begin{pmatrix}h_{t-1}\\ x_t\end{pmatrix} + b_h\right)$$

$$c_t = f_t \odot c_{t-1} + i_t \odot g_t$$

$$h_t = o_t \odot \tanh(c_t)$$



Multi-Layer RNN

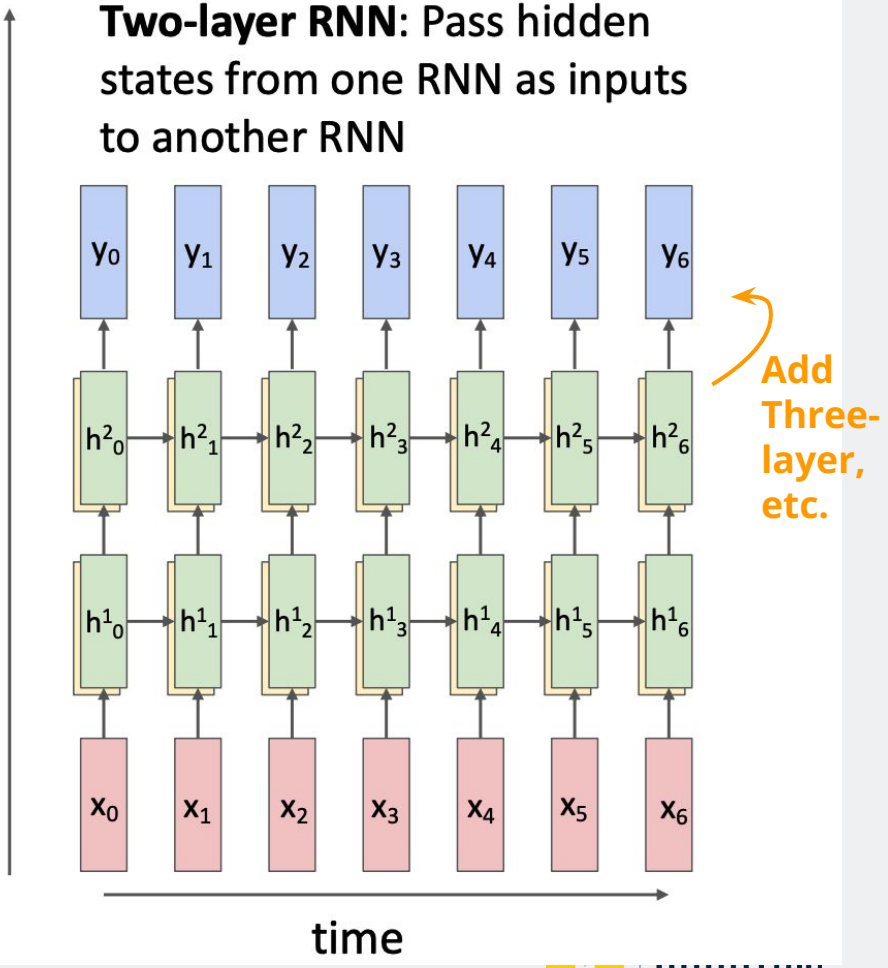
$$h_t^\ell = \tanh\left(W\begin{pmatrix} h_{t-1}^\ell \\ h_t^{\ell-1} \end{pmatrix} + b_h^\ell\right)$$

LSTM:

$$\begin{pmatrix} i_t^\ell \\ f_t^\ell \\ o_t^\ell \\ g_t^\ell \end{pmatrix} = \begin{pmatrix} \sigma \\ \sigma \\ \sigma \\ \tanh \end{pmatrix}\left(W\begin{pmatrix} h_{t-1}^\ell \\ h_t^{\ell-1} \end{pmatrix} + b_h^\ell\right)$$

$$c_t^\ell = f_t^\ell \odot c_{t-1}^\ell + i_t^\ell \odot g_t^\ell$$

$$h_t^\ell = o_t^\ell \odot \tanh(c_t^\ell)$$



Vanilla RNN - 112 Lines of Python Code

Minimal character-level language model with a Vanilla Recurrent Neural Network, in Python/numpy

min-char-rnn.py

```python
"""
Minimal character-level Vanilla RNN model. Written by Andrej Karpathy (@karpathy)
BSD License
"""
import numpy as np

# data I/O
data = open('input.txt', 'r').read() # should be simple plain text file
chars = list(set(data))
data_size, vocab_size = len(data), len(chars)
print 'data has %d characters, %d unique.' % (data_size, vocab_size)
char_to_ix = { ch:i for i,ch in enumerate(chars) }
ix_to_char = { i:ch for i,ch in enumerate(chars) }

# hyperparameters
hidden_size = 100 # size of hidden layer of neurons
seq_length = 25 # number of steps to unroll the RNN for
learning_rate = 1e-1

# model parameters
Wxh = np.random.randn(hidden_size, vocab_size)*0.01 # input to hidden
Whh = np.random.randn(hidden_size, hidden_size)*0.01 # hidden to hidden
Why = np.random.randn(vocab_size, hidden_size)*0.01 # hidden to output
bh = np.zeros((hidden_size, 1)) # hidden bias
by = np.zeros((vocab_size, 1)) # output bias

def lossFun(inputs, targets, hprev):
  """
  inputs,targets are both list of integers.
  hprev is Hx1 array of initial hidden state
  returns the loss, gradients on model parameters, and last hidden state
  """
  xs, hs, ys, ps = {}, {}, {}, {}
  hs[-1] = np.copy(hprev)
  loss = 0
```

https://gist.github.com/karpathy/d4dee566867f8291f086

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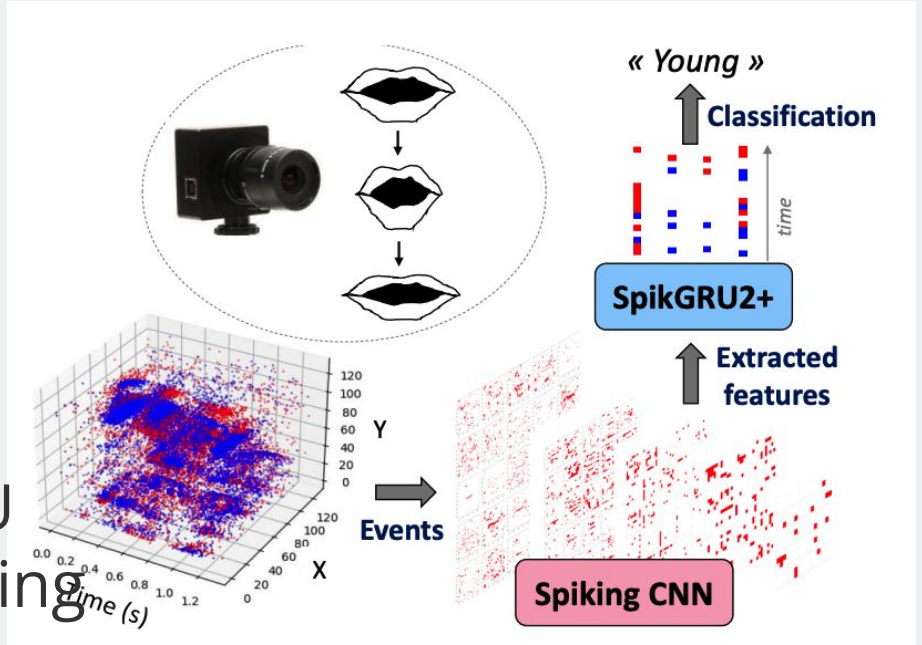
Gated Recurrent Unit (GRU)

$$r_t = \sigma(W_{xr}x_t + W_{hr}h_{t-1} + b_r)$$
$$z_t = \sigma(W_{xz}x_t + W_{hz}h_{t-1} + b_z)$$
$$\tilde{h}_t = \tanh(W_{xh}x_t + W_{hh}(r_T \odot h_{t-1}) + b_h)$$
$$h_t = z_t \odot h_{t-1} + (1 - z_t) \odot \tilde{h}_t$$

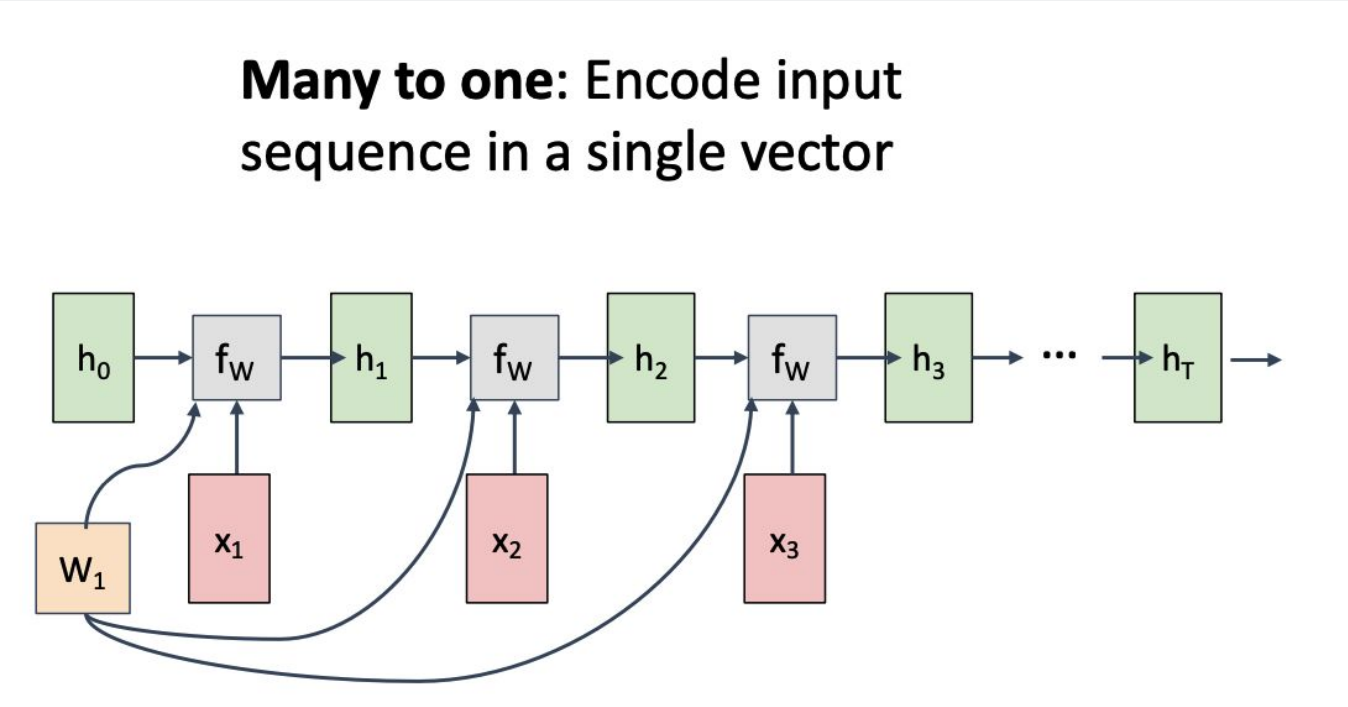


E.g., CVPR 2024
Spiking Neural Networks + GRU
for speech recognition/lip reading

Cho et al “Learning phrase representations using RNN encoder-decoder for statistical machine translation”, 2014
https://arxiv.org/abs/1406.1078

https://openaccess.thecvf.com/content/CVPR2024W/EVW/papers/Dampfhoffer_Neuromorphic_Lip-Reading_with_Signed_Spiking_Gated_Recurrent_Units_CVPRW_2024_paper.pdf
Spike GPT https://arxiv.org/pdf/2302.13939

Seq2Seq: Sequence to Sequence

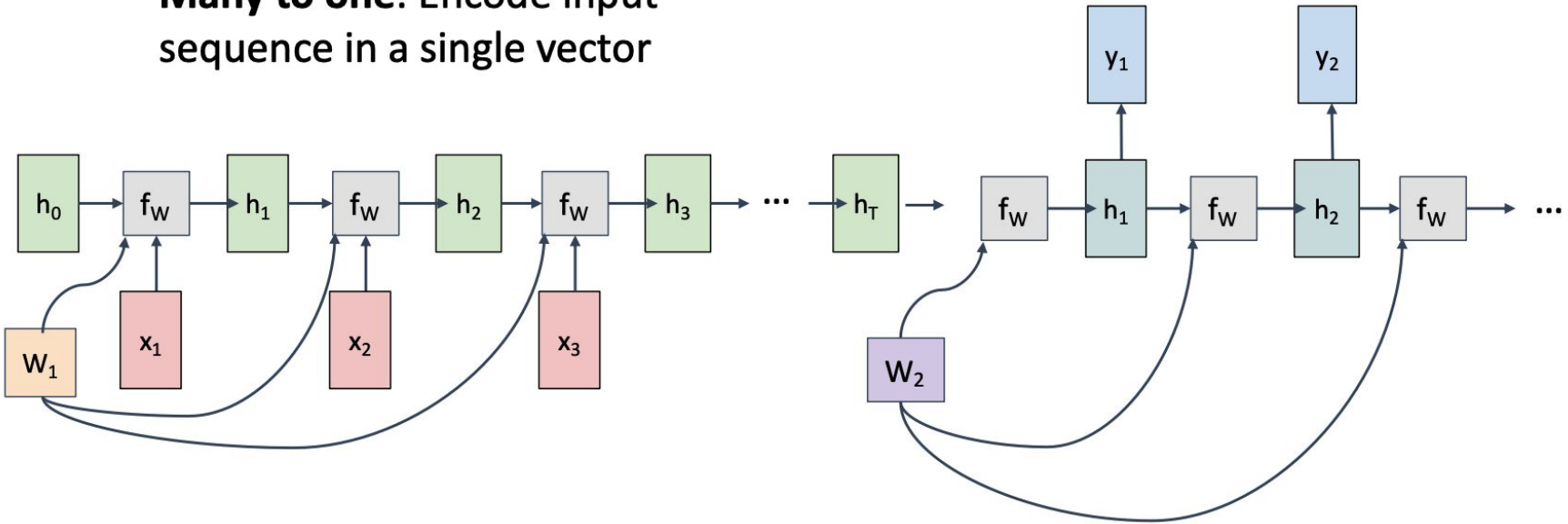


https://proceedings.neurips.cc/paper_files/paper/2014/file/5a18e133cbf9f257297f410bb7eca942-Paper.pdf

Seq2Seq: Sequence to Sequence

One to many: Produce output sequence from single input vector

Many to one: Encode input sequence in a single vector



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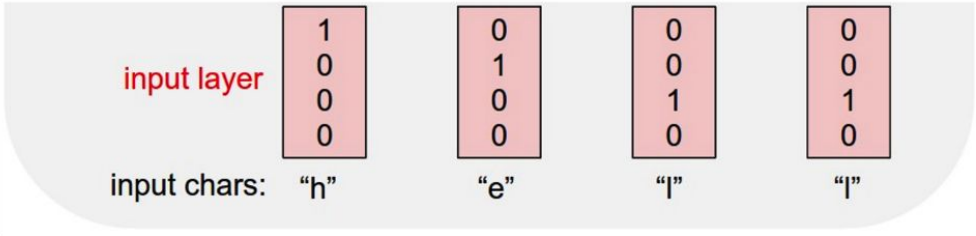
Example: Language Modeling

Given characters 1, 2, …, t-1,
model predicts character t

Training sequence: ”hello”

Vocabulary: [h, e, l, o]



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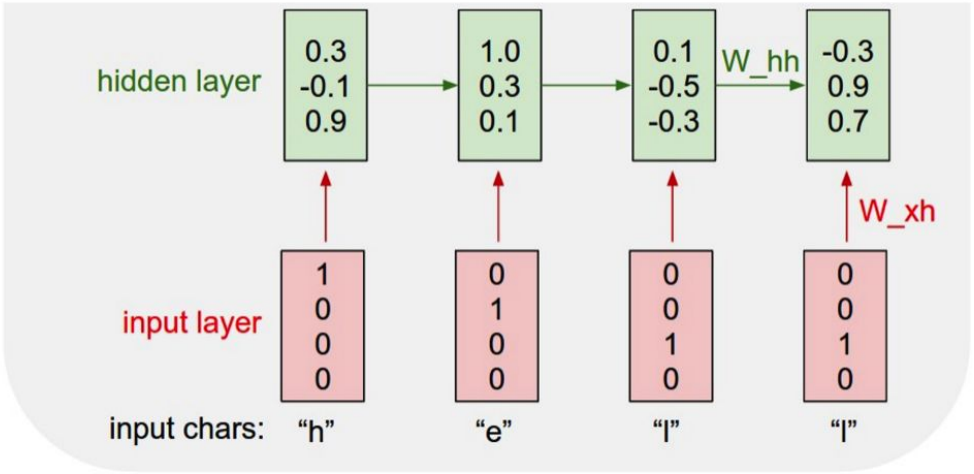

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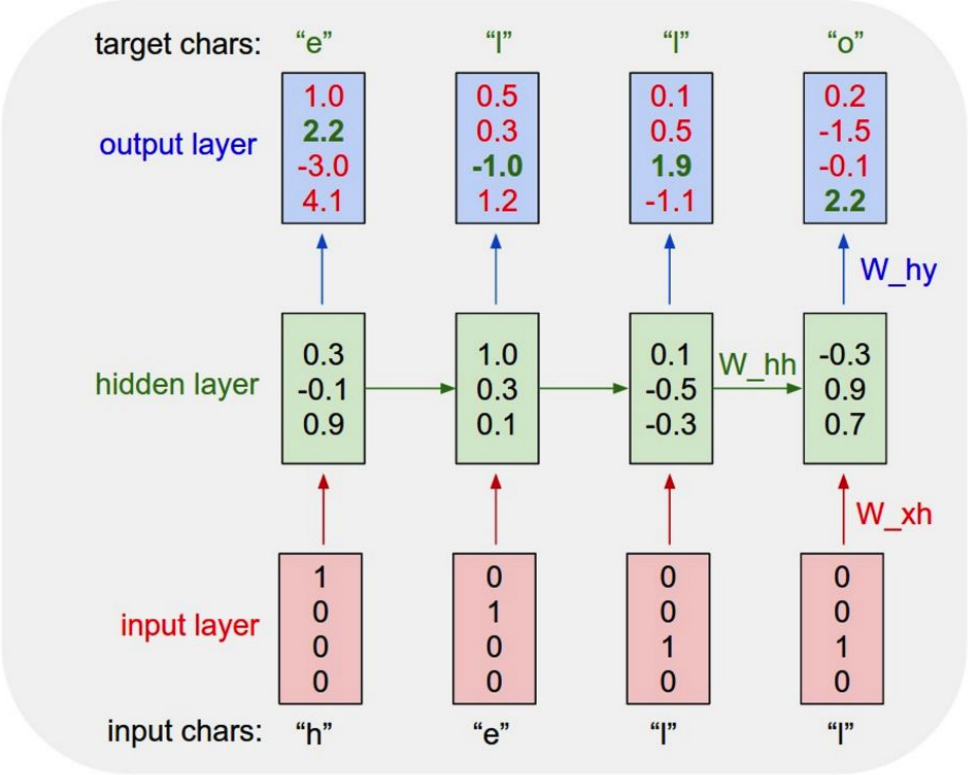

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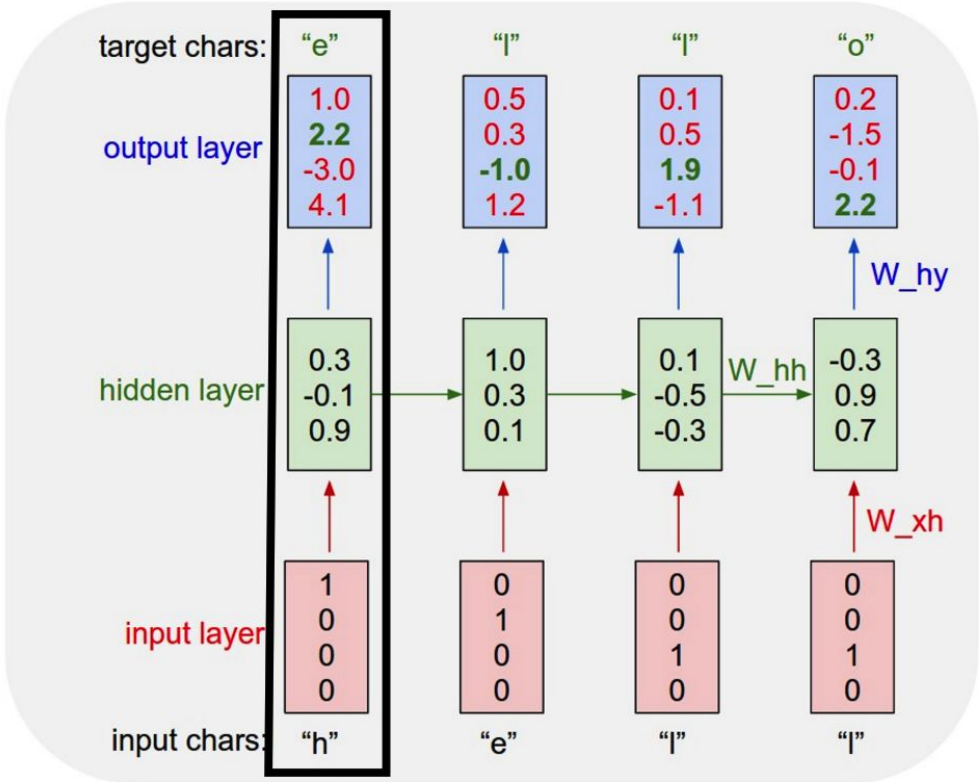

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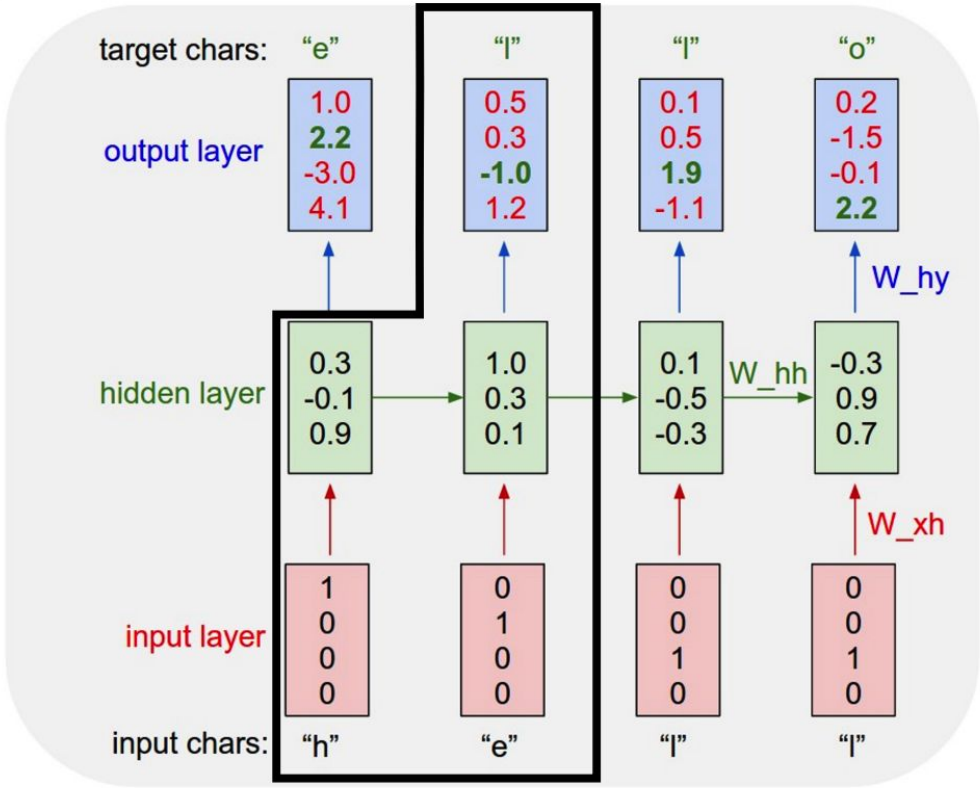

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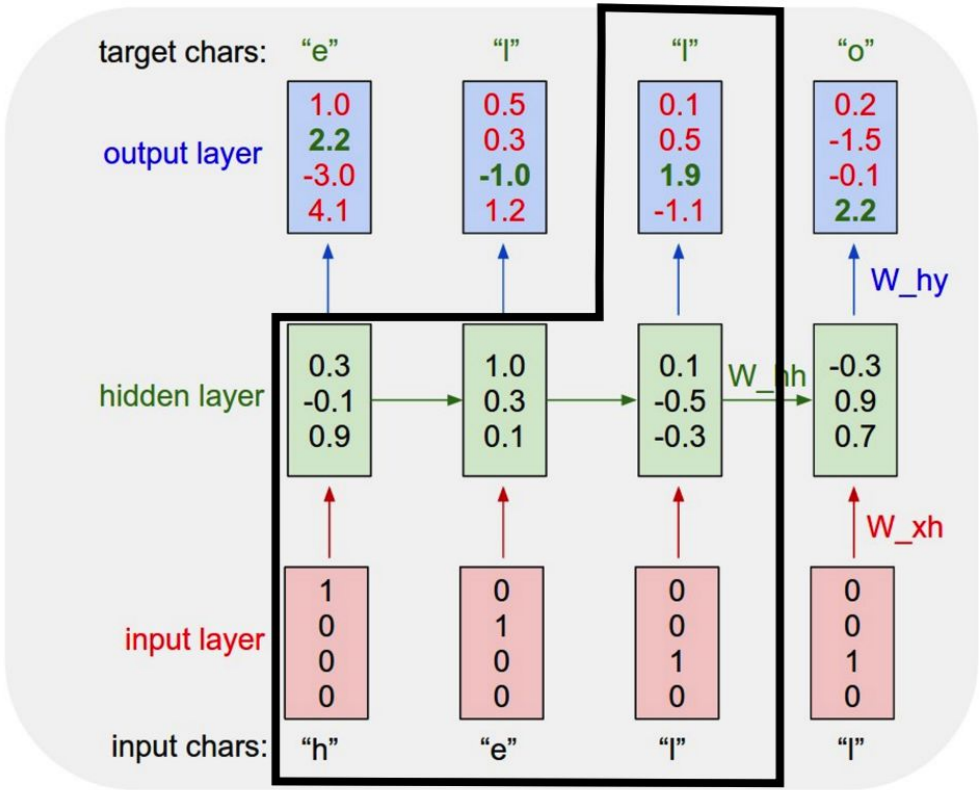

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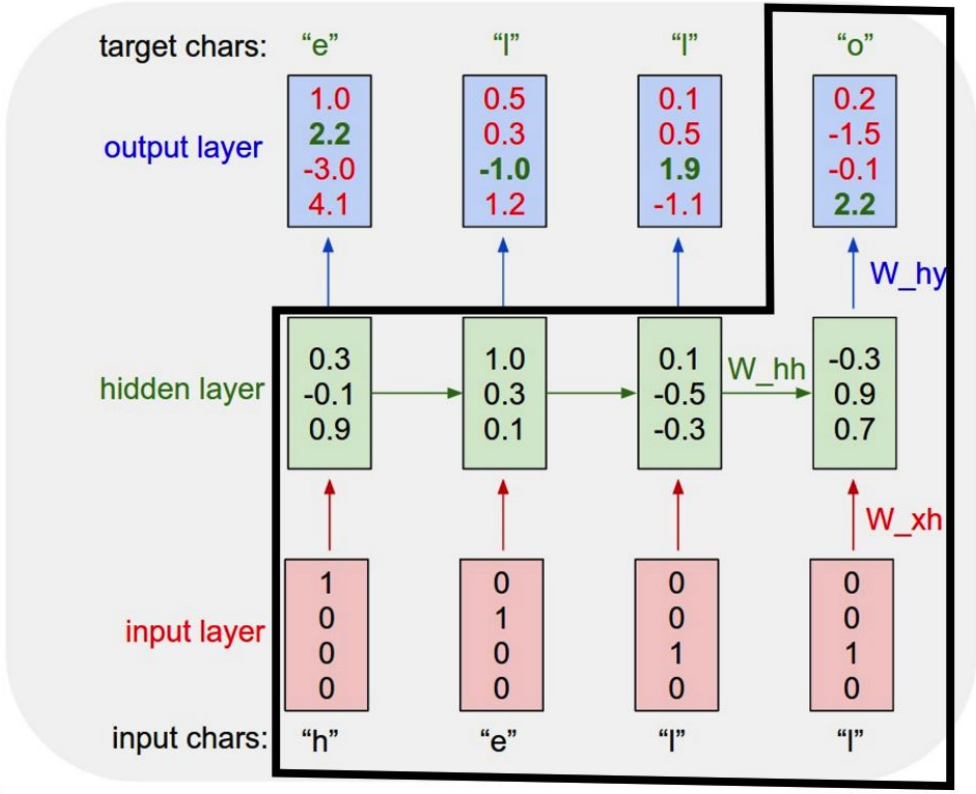
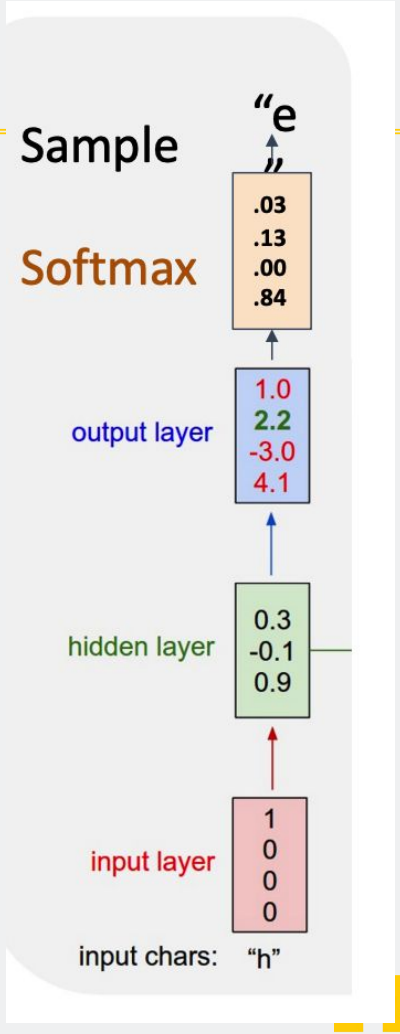


Example: Language Modeling

At test-time, **generate** new text: sample characters one at a time, feed back to model

Training sequence: ”hello”

Vocabulary: [h, e, l, o]



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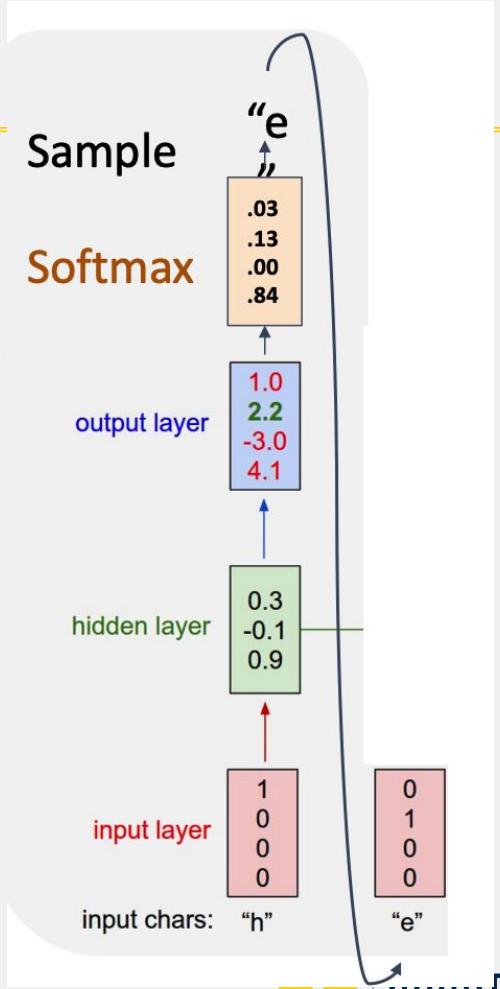

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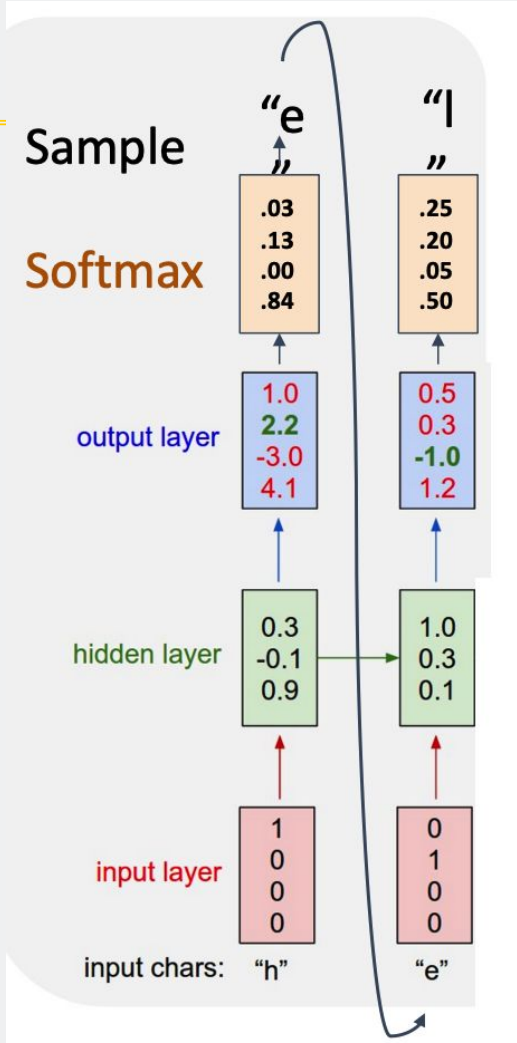
Training sequence: ”hello”

Vocabulary: [h, e, l, o]

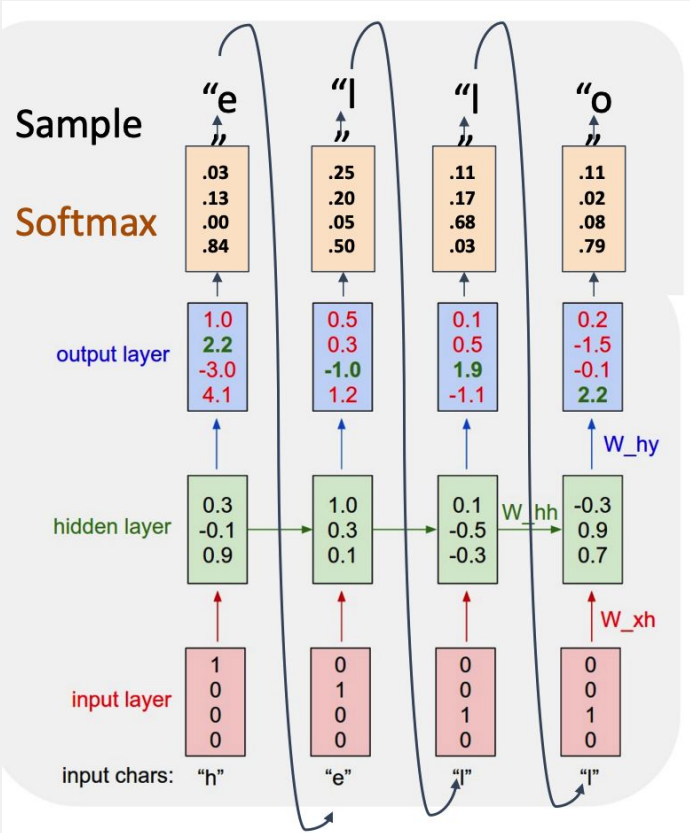


Example: Language Modeling

At test-time, **generate** new text: sample characters one at a time, feed back to model

Training sequence: ”hello”

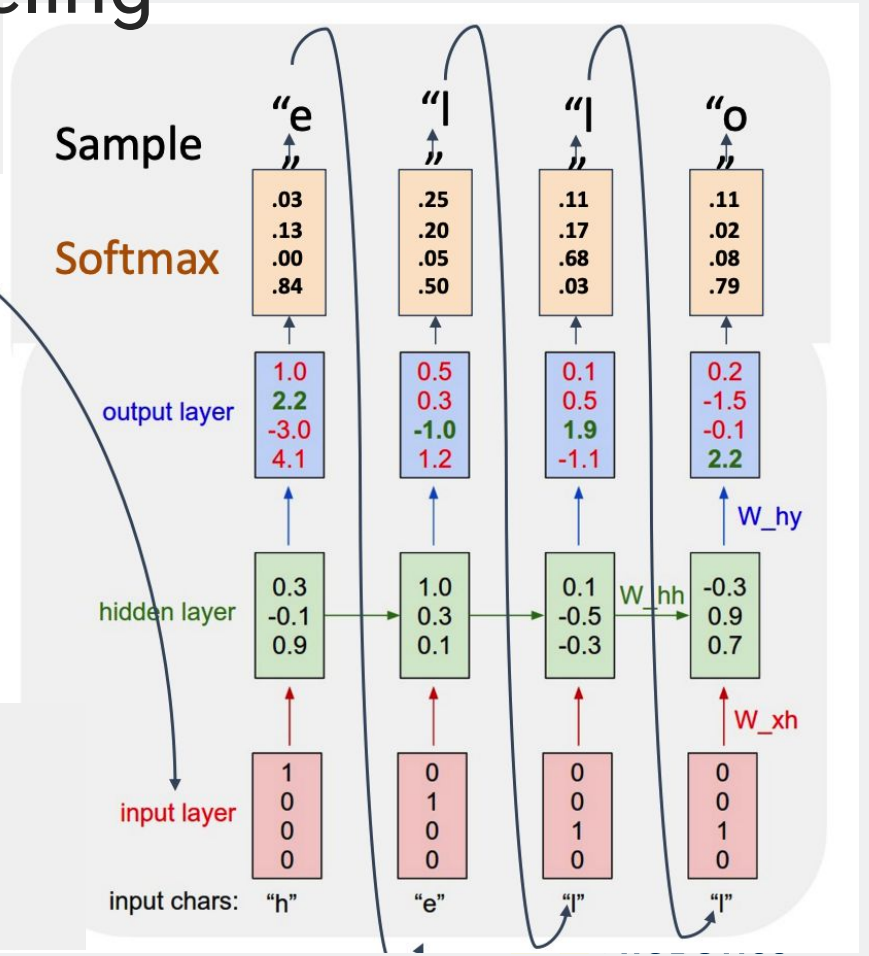
Vocabulary: [h, e, l, o]



Example: Language Modeling

So far: encode inputs as **one-hot-vector**

$$\begin{bmatrix} w_{11} & w_{12} & w_{13} & w_{14} \\ w_{21} & w_{22} & w_{23} & w_{14} \\ w_{31} & w_{32} & w_{33} & w_{14} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} w_{11} \\ w_{21} \\ w_{31} \end{bmatrix}$$



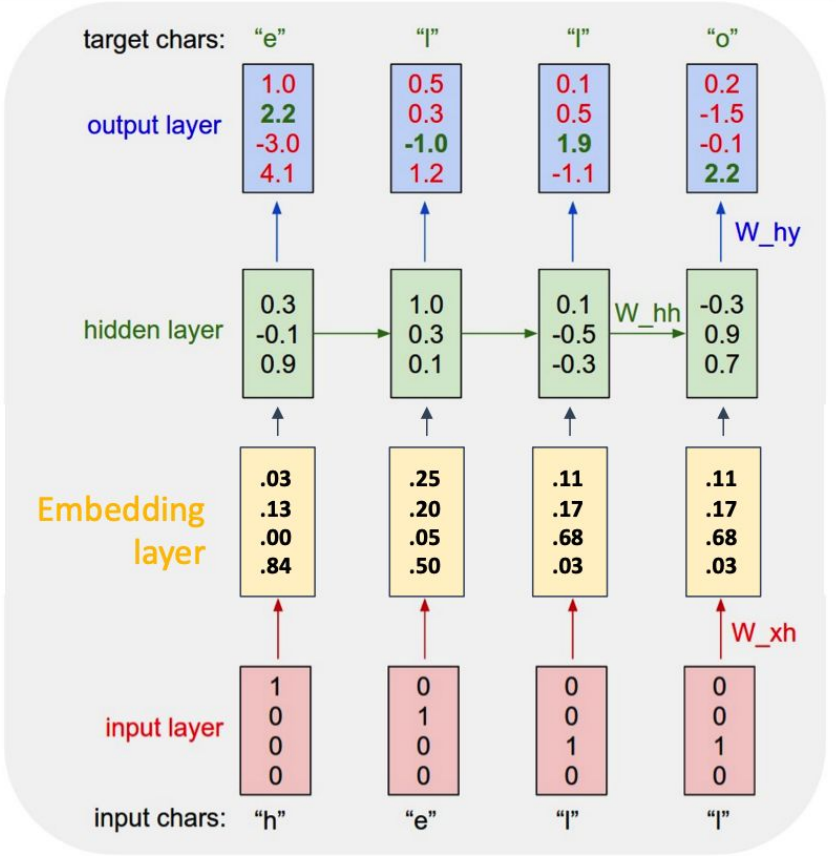
Example: Language Modeling

*NN likes dense real-valued vectors

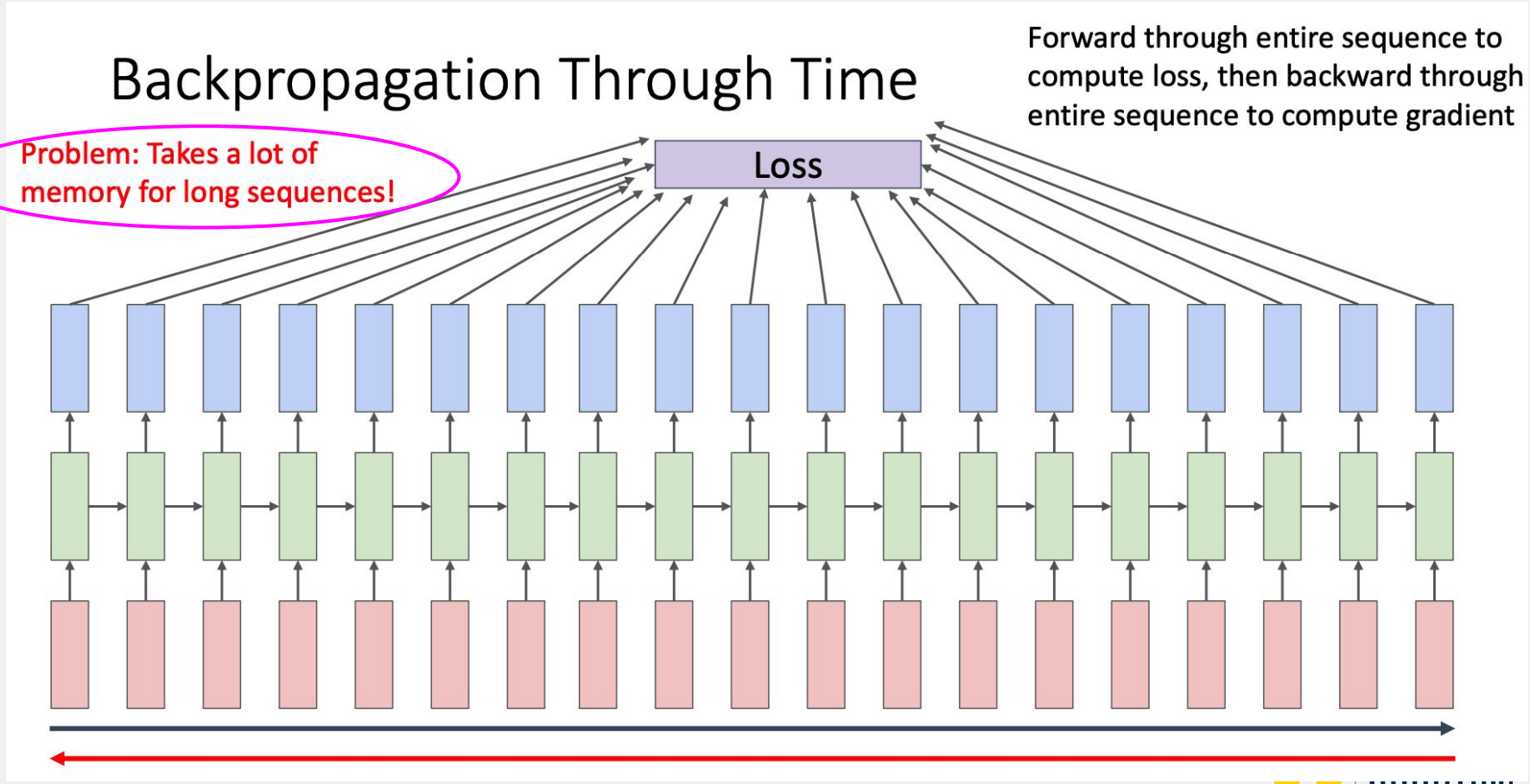
So far: encode inputs as **one-hot-vector**

$$\begin{bmatrix} w_{11} & w_{12} & w_{13} & w_{14} \\ w_{21} & w_{22} & w_{23} & w_{14} \\ w_{31} & w_{32} & w_{33} & w_{14} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} w_{11} \\ w_{21} \\ w_{31} \end{bmatrix}$$

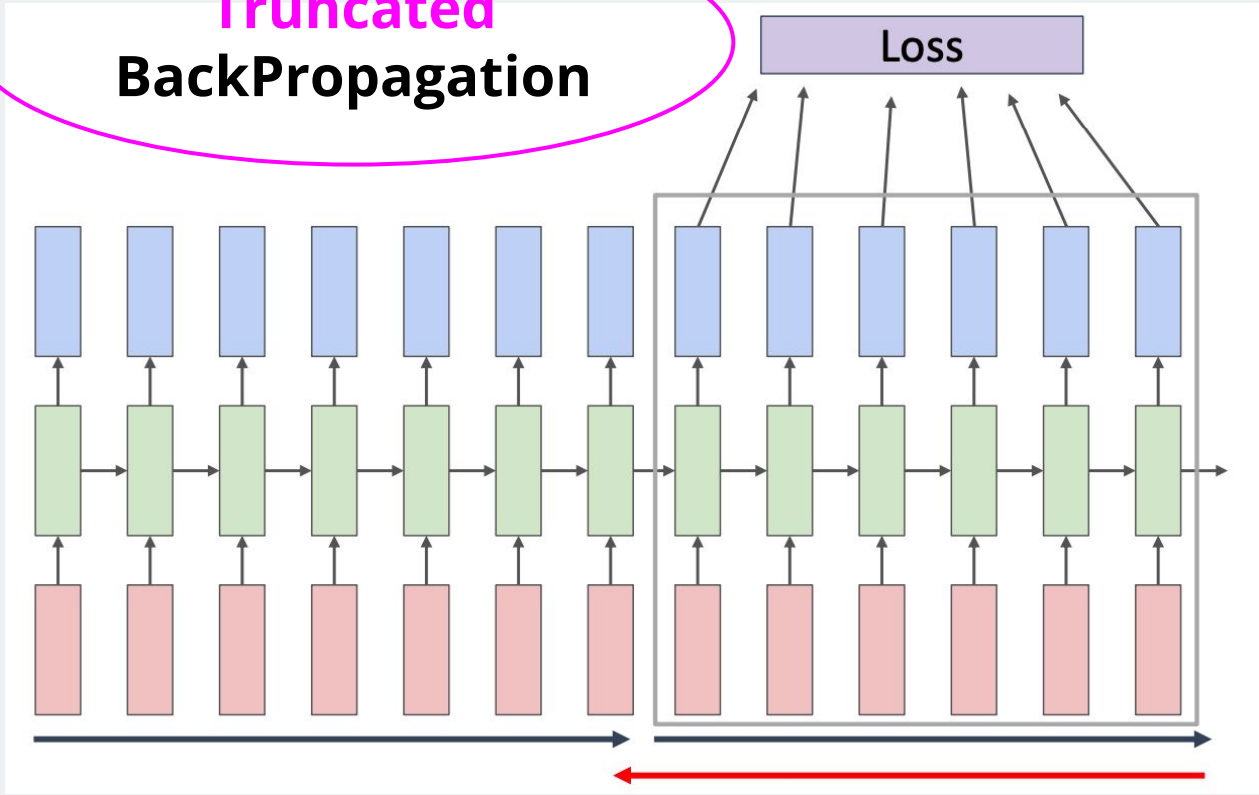
Matrix multiply with a one-hot vector just extracts a column from the weight matrix. Often extract this into a separate **embedding** layer



Example: Language Modeling (Backprop)



Example: Language Modeling (Backprop)



- Run forward and backward through chunks of the sequence instead of whole sequence
- Carry hidden states forward in time forever, but only backpropagate for some smaller number of steps

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Limitations of RNNs

- Modeling long-range dependencies limited by vanishing gradient
- Computational and memory efficiency, especially for long sequences
- Parallelization of layers that depend on sequential information

Modeling Long-Range Dependencies

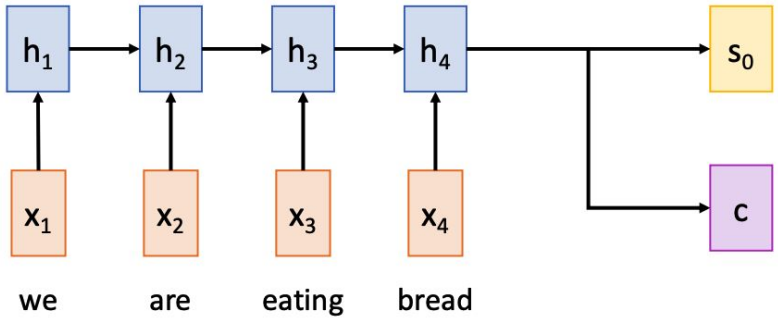
One Example: Language Translation

Input: Sequence $x_1, \ldots x_T$

Output: Sequence $y_1, \ldots, y_{T'}$

Encoder: $h_t = f_W(x_t, h_{t-1})$

From final hidden state predict:
Initial decoder state s_0
Context vector c (often $c=h_T$)



Recall: Seq2Seq + RNN

Sutskever et al, “Sequence to sequence learning with neural networks”, NeurIPS 2014
https://papers.nips.cc/paper_files/paper/2014/file/5a18e133cbf9f257297f410bb7eca942-Paper.pdf

Modeling Long-Range Dependencies

One Example: Language Translation

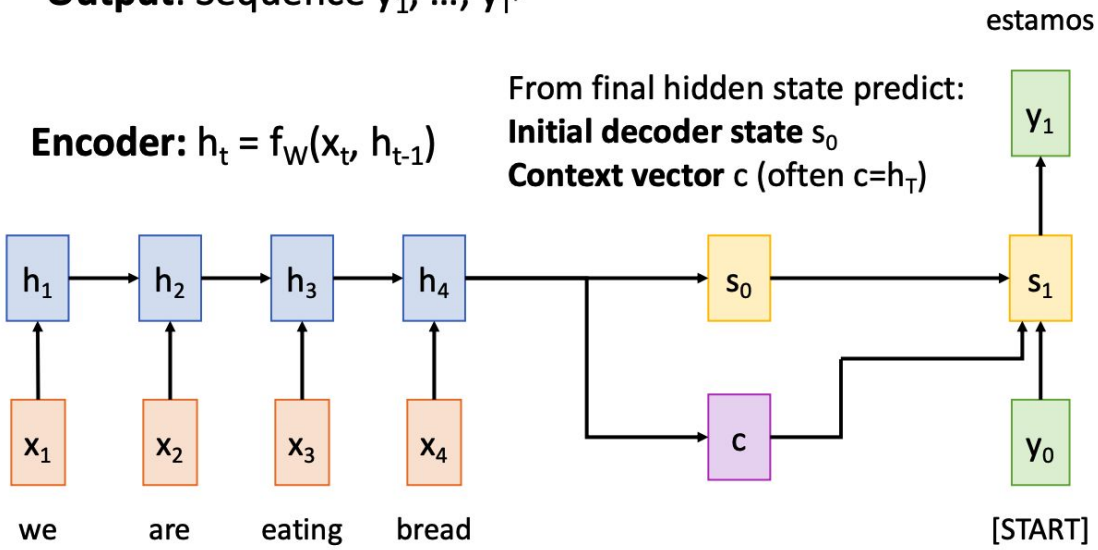
Input: Sequence $x_1, \ldots x_T$

Output: Sequence $y_1, \ldots, y_{T'}$

Decoder: $s_t = g_U(y_{t-1}, s_{t-1}, c)$

Encoder: $h_t = f_W(x_t, h_{t-1})$

From final hidden state predict:
Initial decoder state s_0
Context vector c (often $c=h_T$)



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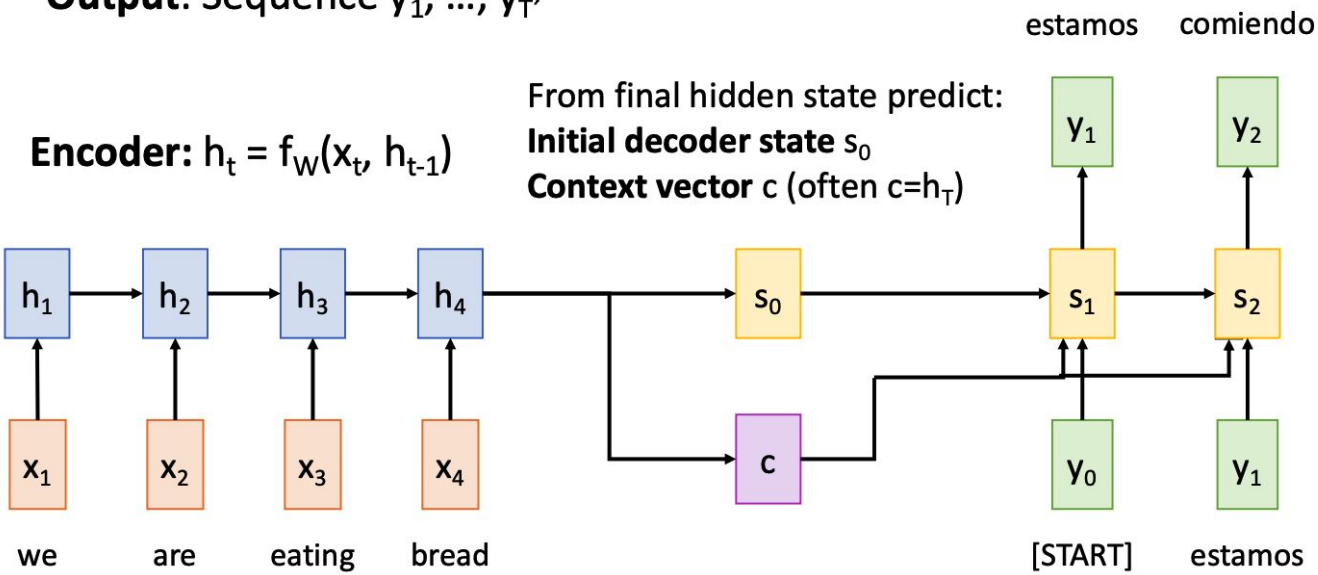

Sutskever et al, "Sequence to sequence learning with neural networks", NeurIPS 2014
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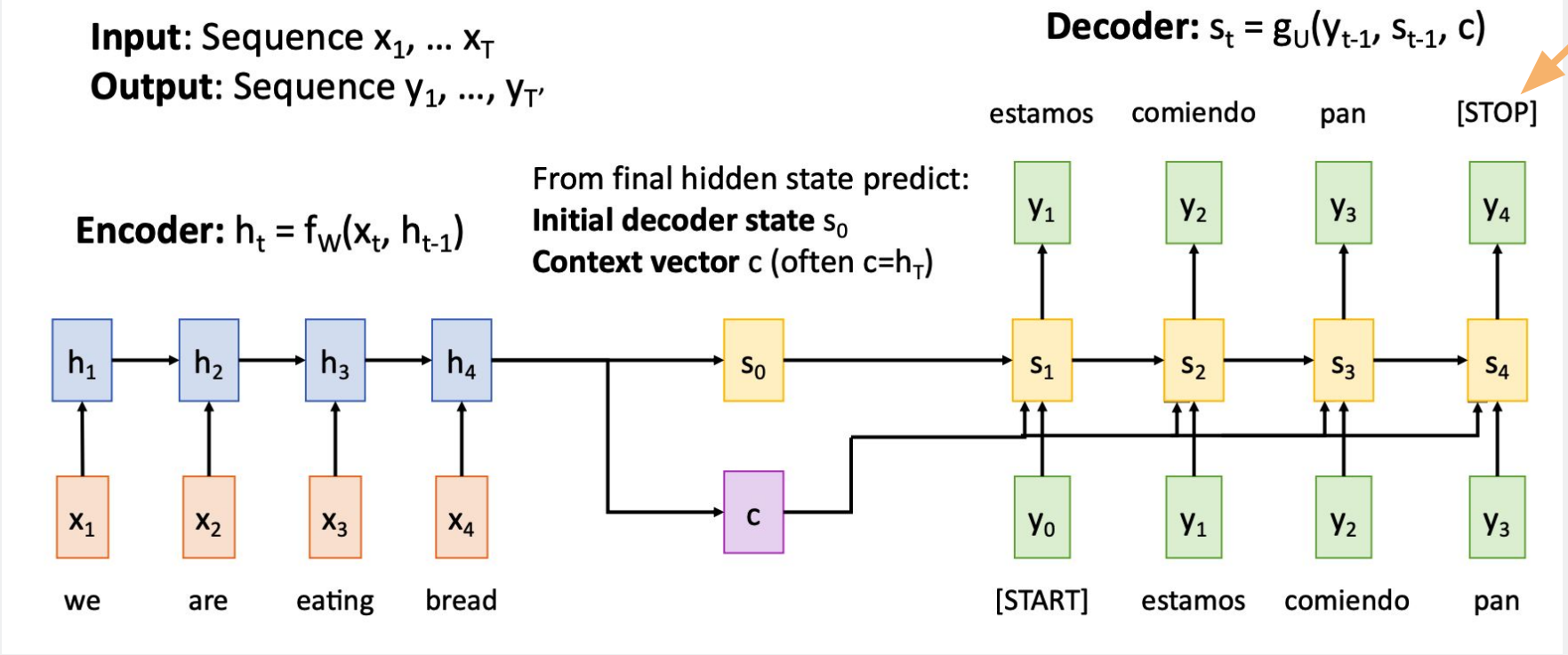
Modeling Long-Range Dependencies

One Example: Language Translation



Sutskever et al, “Sequence to sequence learning with neural networks”, NeurIPS 2014
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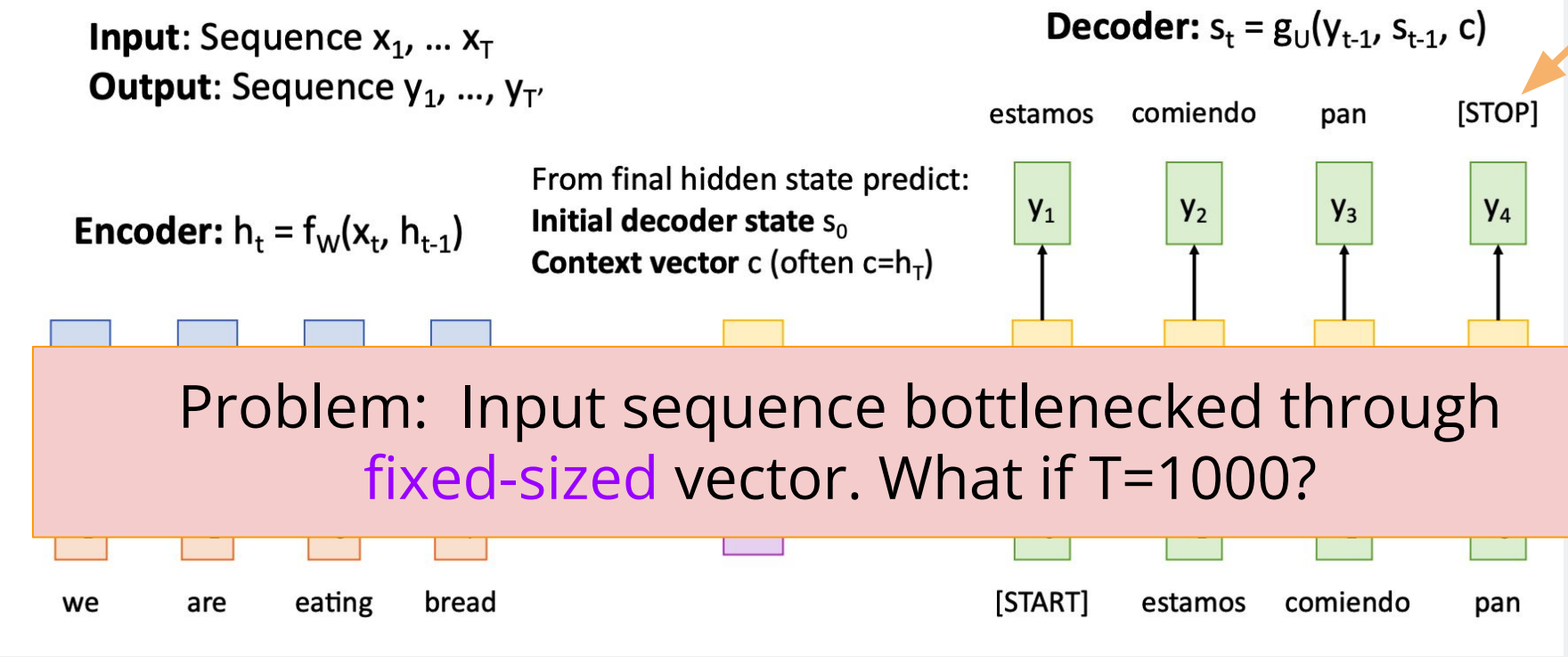
Modeling Long-Range Dependencies

One Example: Language Translation

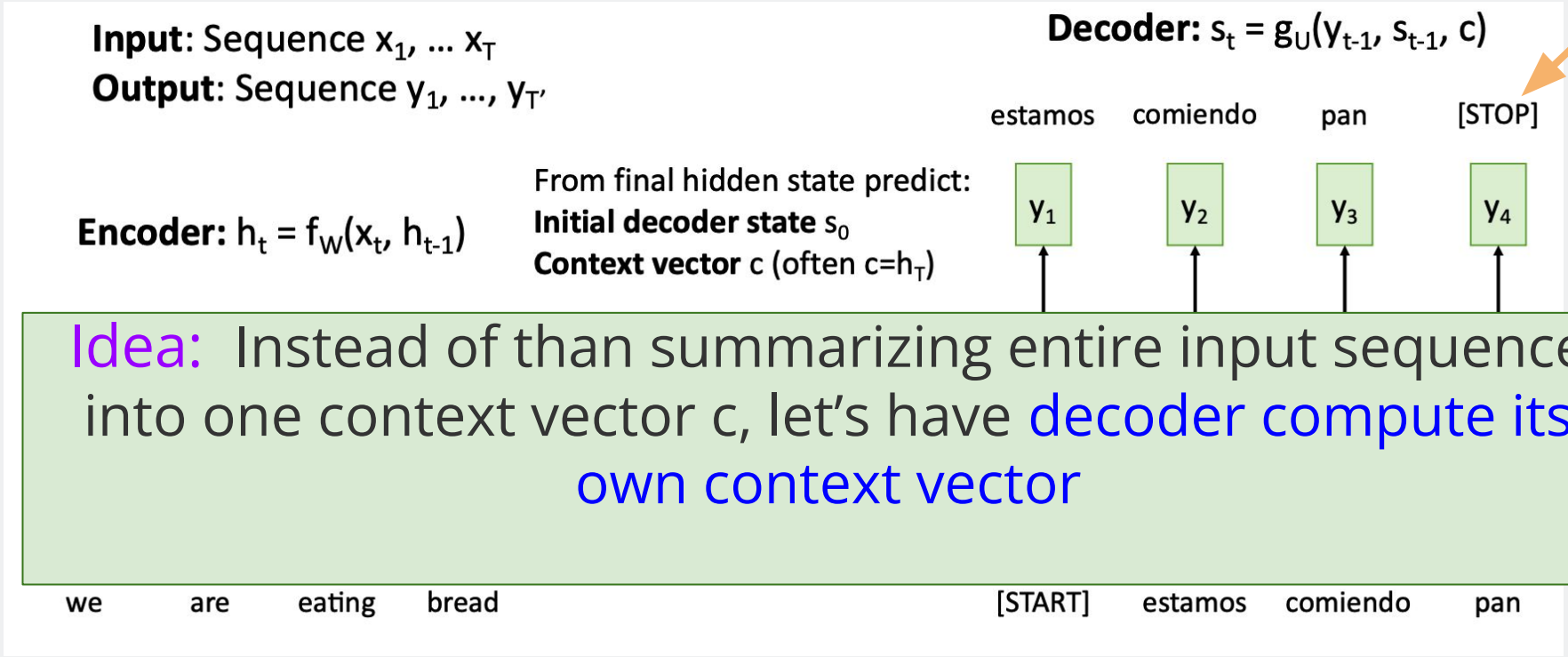


Problem: Input sequence bottlenecked through fixed-sized vector. What if T=1000?

Sutskever et al, “Sequence to sequence learning with neural networks”, NeurIPS 2014
https://papers.nips.cc/paper_files/paper/2014/file/5a18e133cbf9f257297f410bb7eca942-Paper.pdf

Modeling Long-Range Dependencies

One Example: Language Translation



Sutskever et al, "Sequence to sequence learning with neural networks", NeurIPS 2014
https://papers.nips.cc/paper_files/paper/2014/file/5a18e133cbf9f257297f410bb7eca942-Paper.pdf

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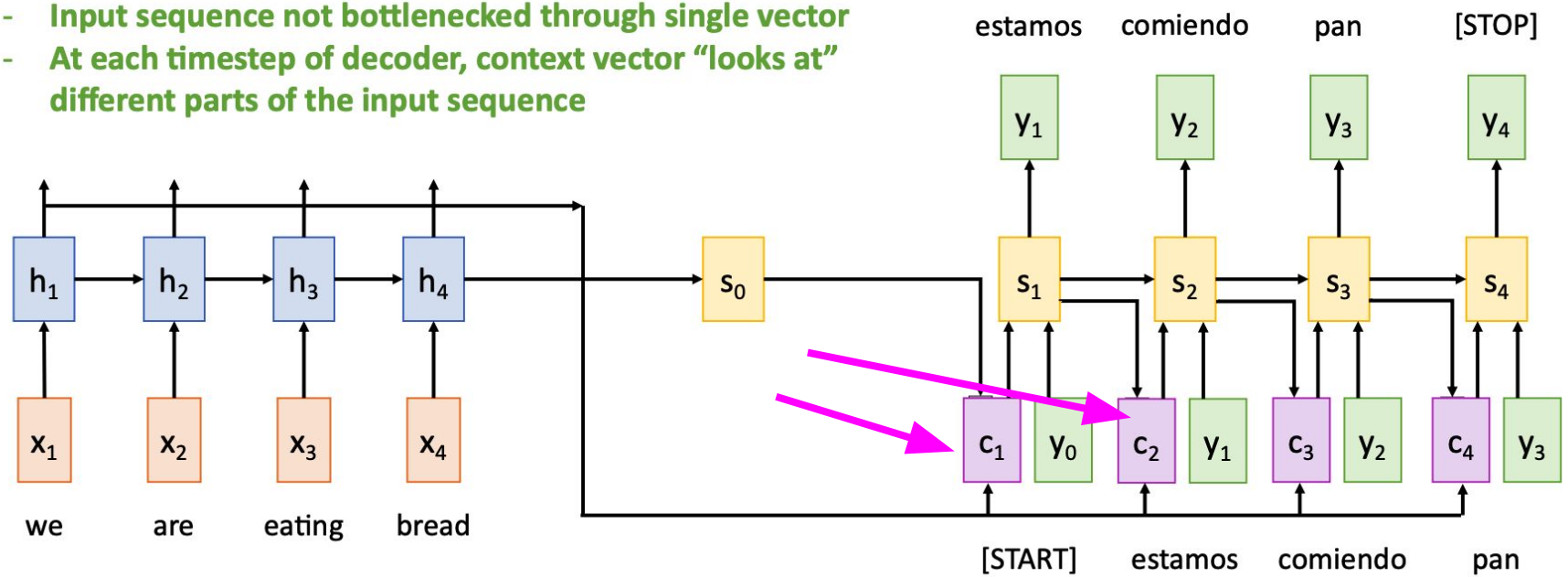
Modeling Long-Range Dependencies

One Example: Language Translation

Use a different context vector in each timestep of decoder

- **Input sequence not bottlenecked through single vector**
- **At each timestep of decoder, context vector "looks at" different parts of the input sequence**



Bahdanau et al, "Neural machine translation by jointly learning to align and translate", ICLR 2015
https://arxiv.org/abs/1409.0473

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