



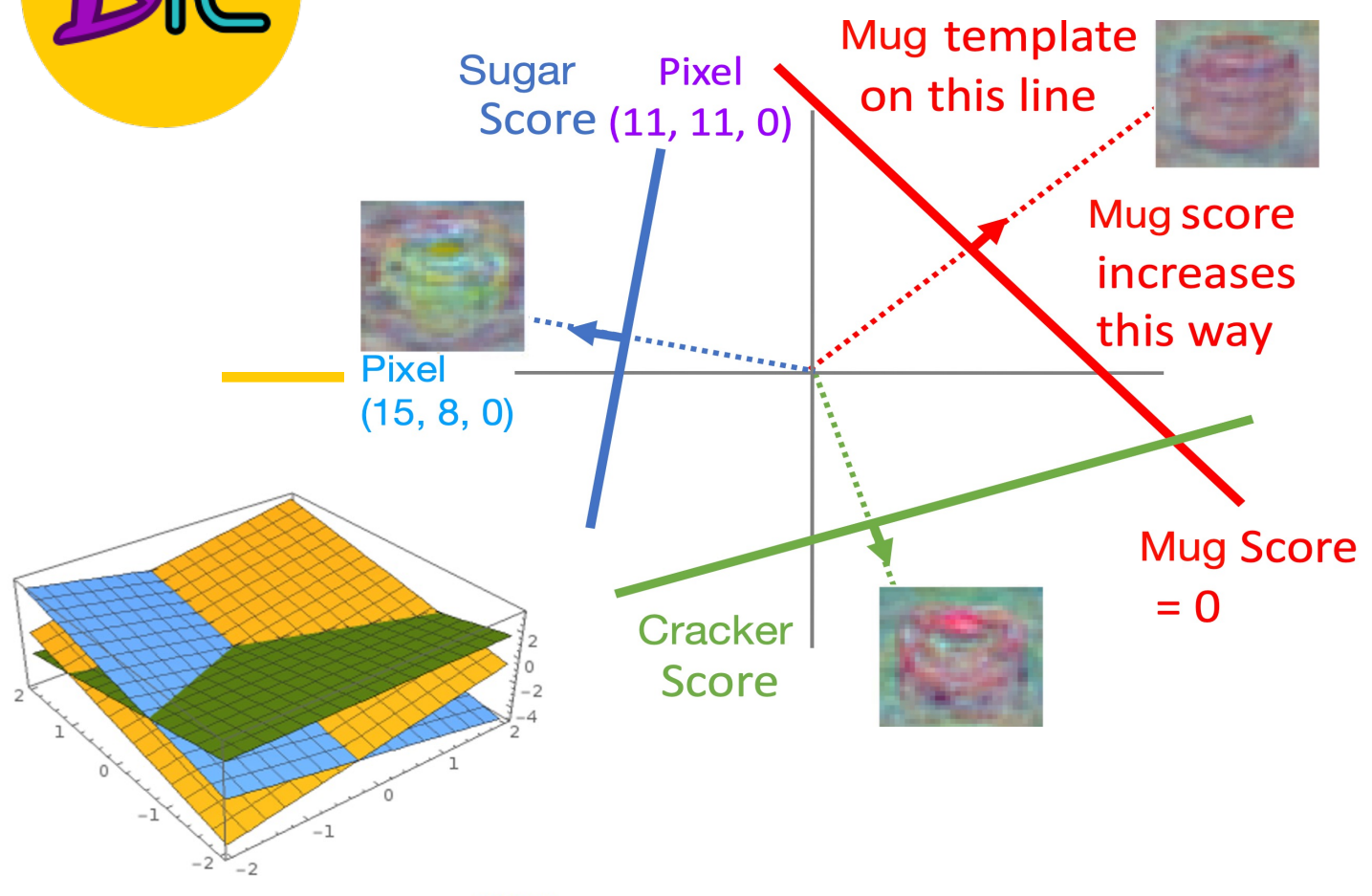
# DEEP ROB

Lecture 6  
Convolutional Neural Networks  
University of Michigan | Department of Robotics

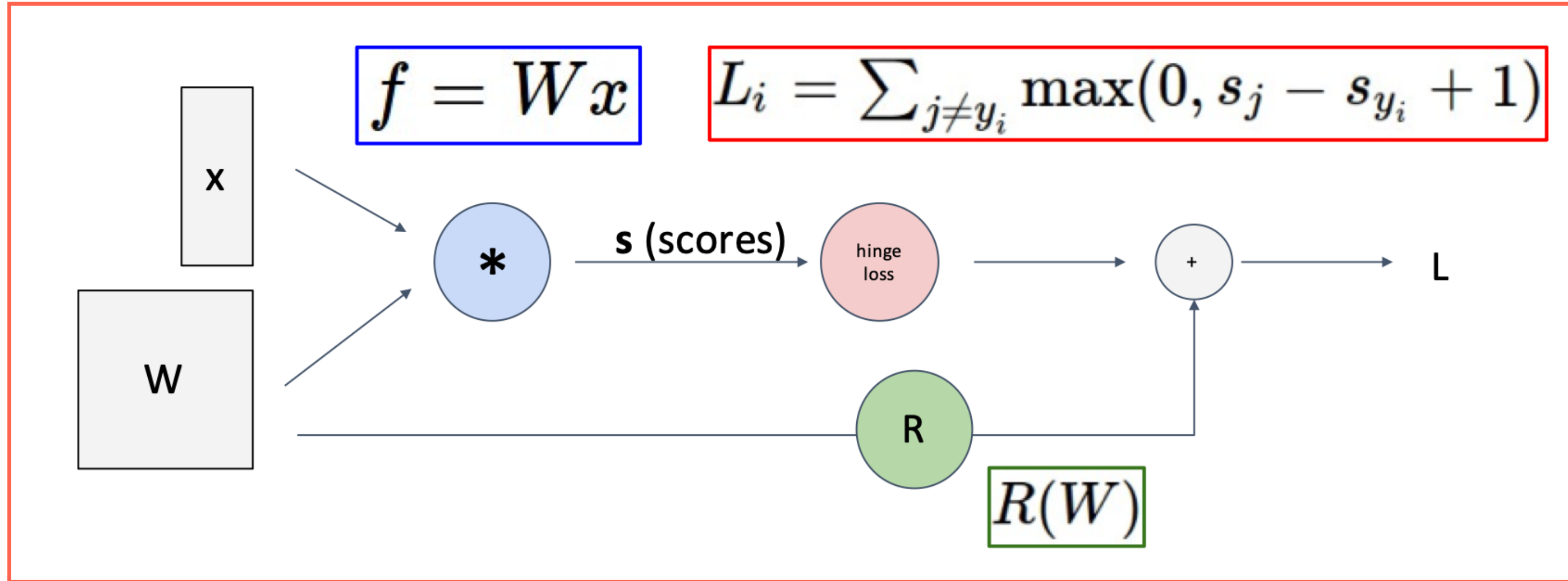
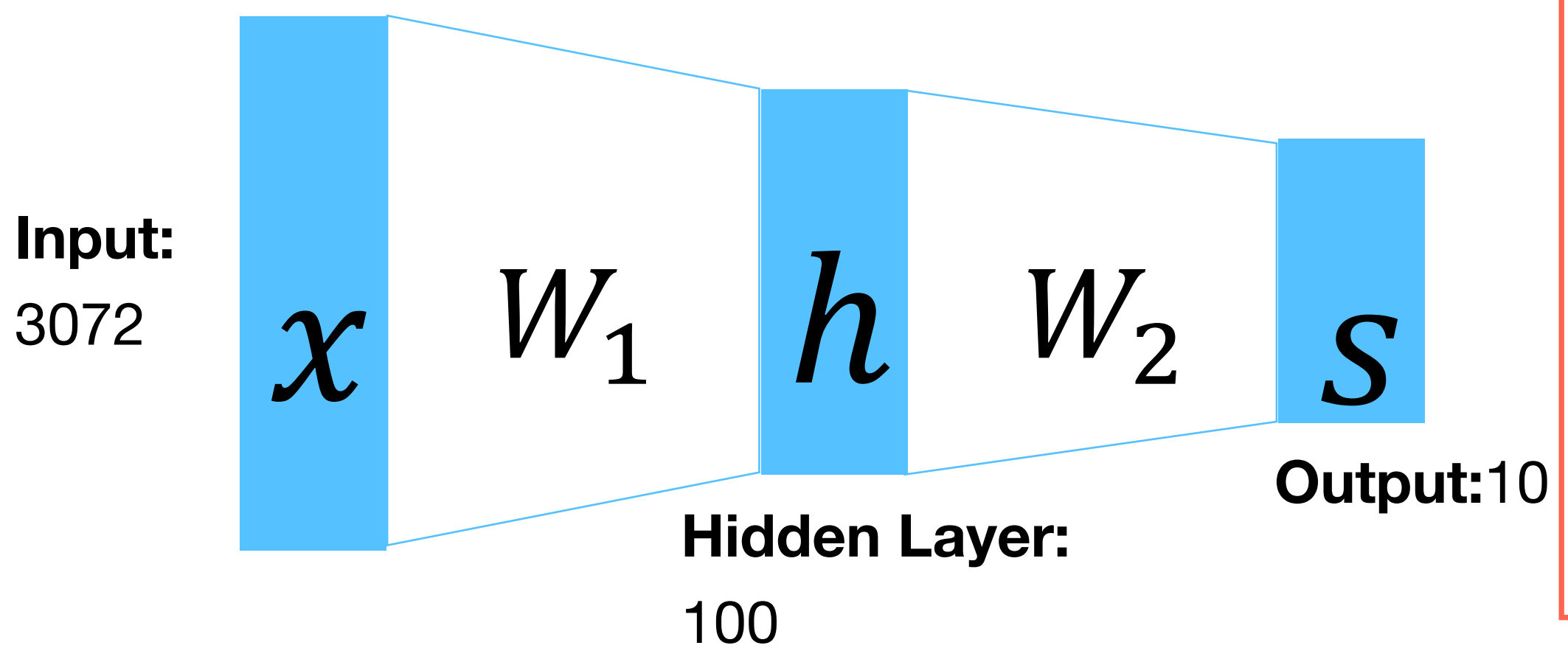




# Recap: Backpropagation



$$f(x) = W_2 \max(0, W_1 x + b_1) + b_2$$



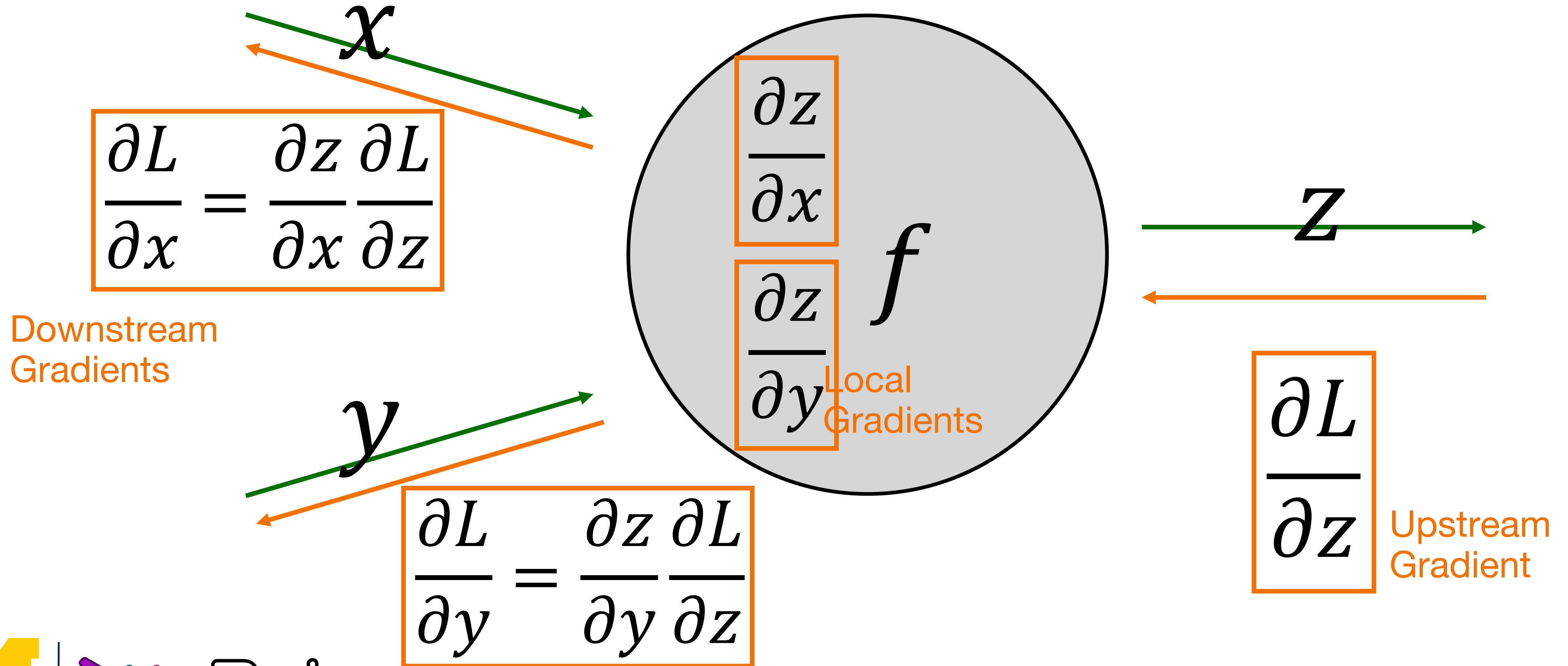
Forward pass

Backward pass (Backprop)

Computational Graph

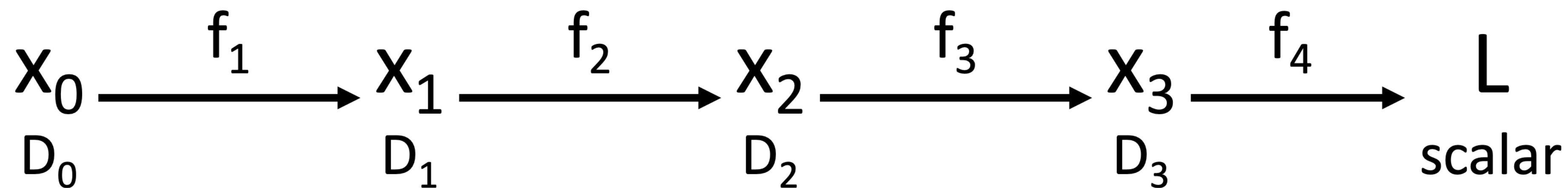


# Recap: Backpropagation





# Recap: “The Chain Rule”

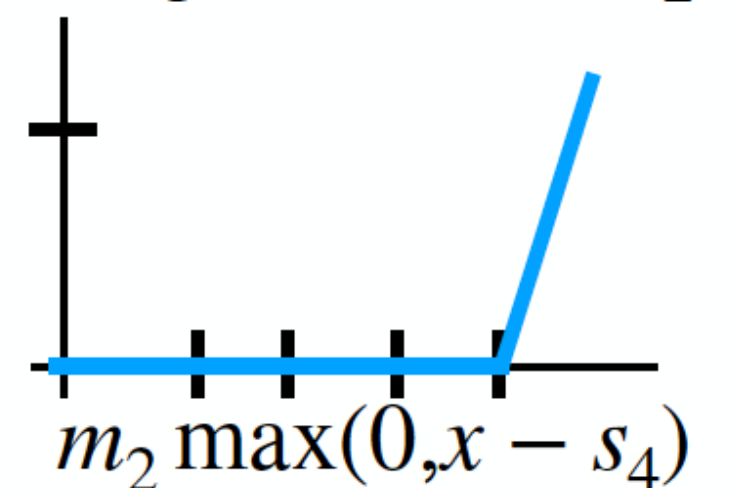
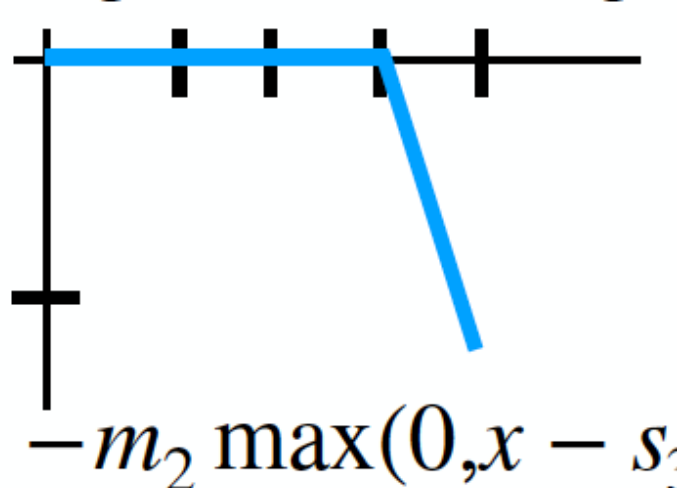
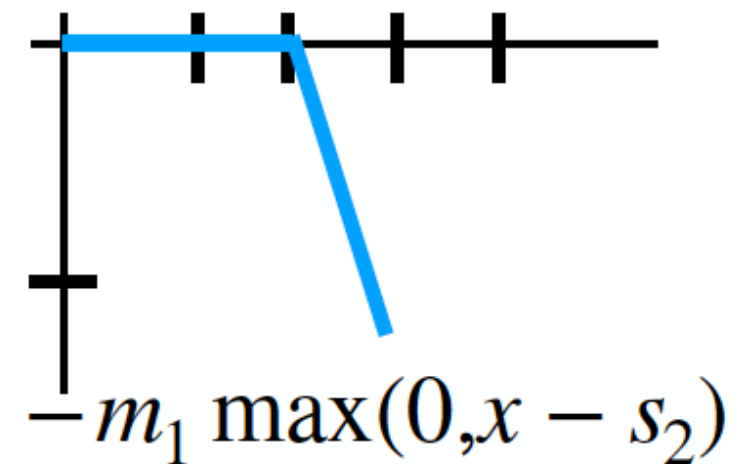
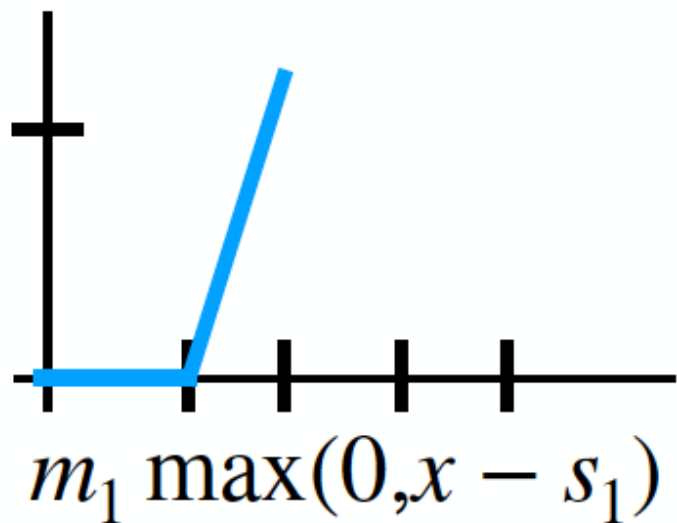
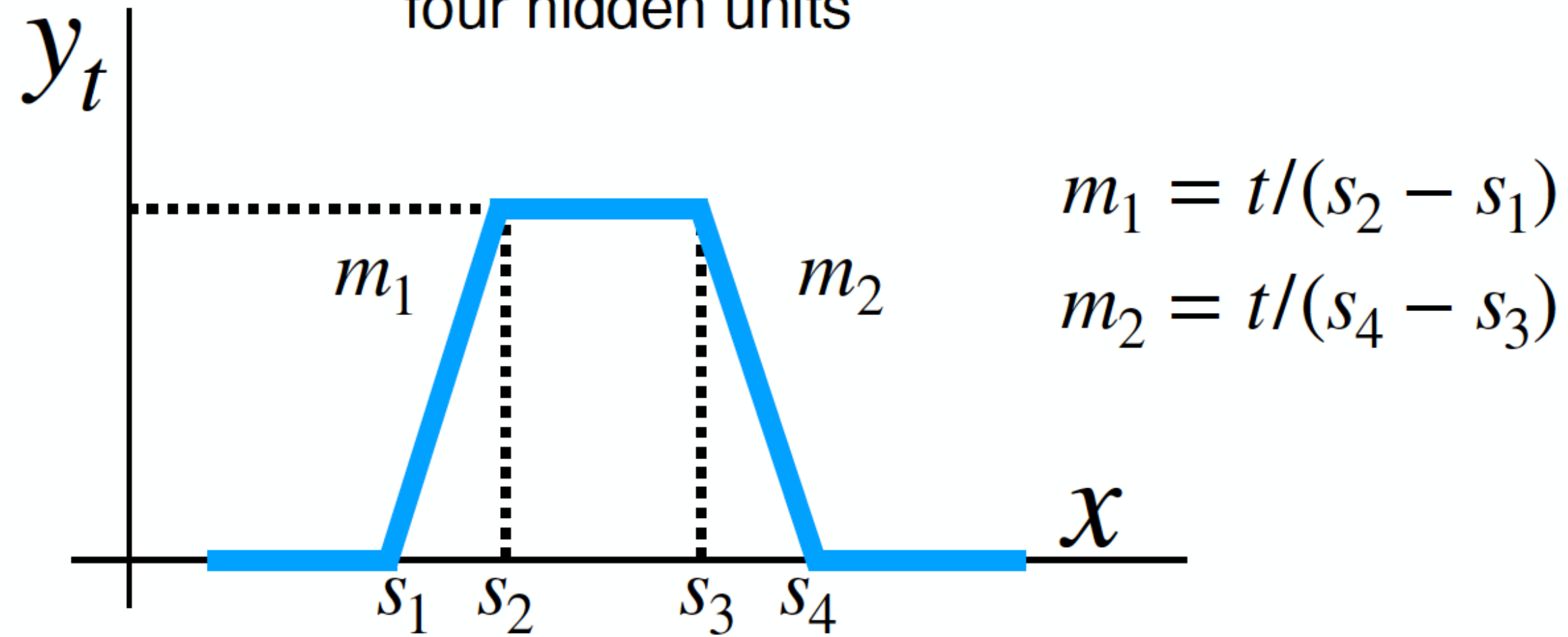


Chain rule  $\frac{\partial L}{\partial x_0} = \left(\frac{\partial x_1}{\partial x_0}\right) \left(\frac{\partial x_2}{\partial x_1}\right) \left(\frac{\partial x_3}{\partial x_2}\right) \left(\frac{\partial L}{\partial x_3}\right)$

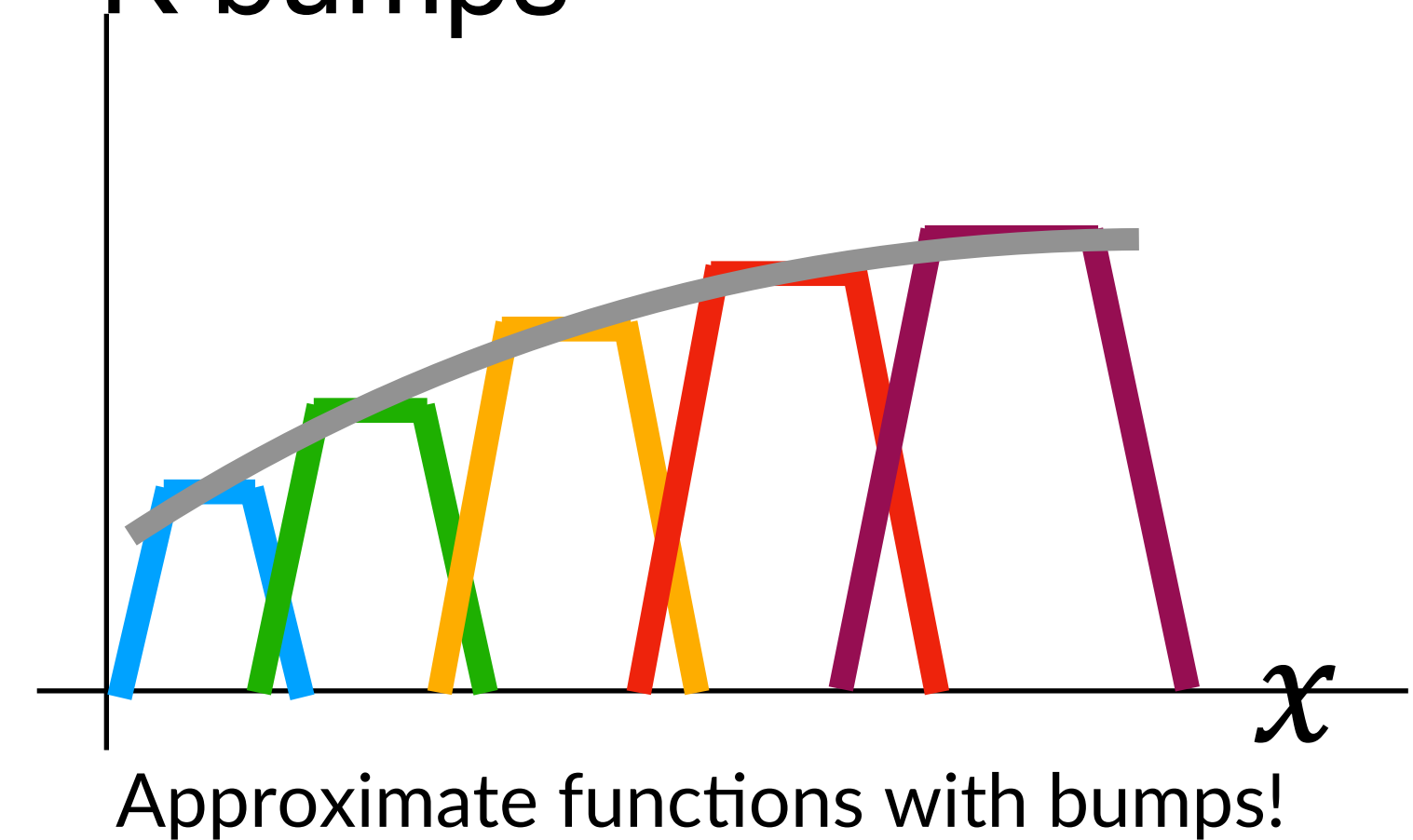


# Recap: Universal Approximation

We can build a “bump function” using four hidden units

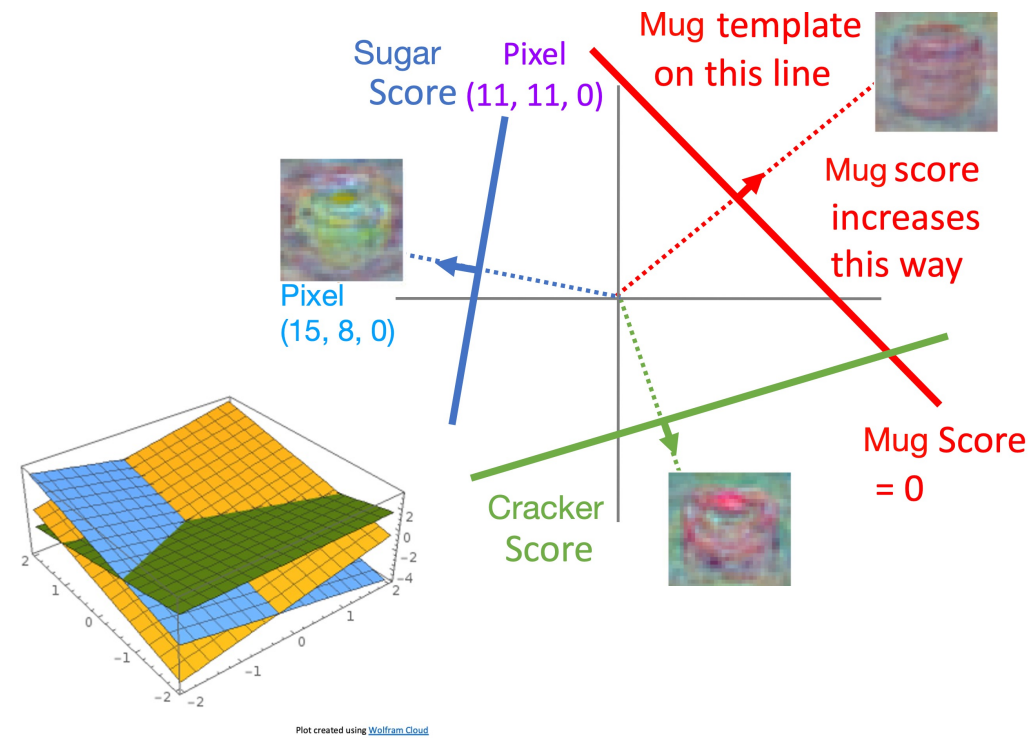


With 4K hidden units we can build a sum of K bumps





# Spatial Structure?

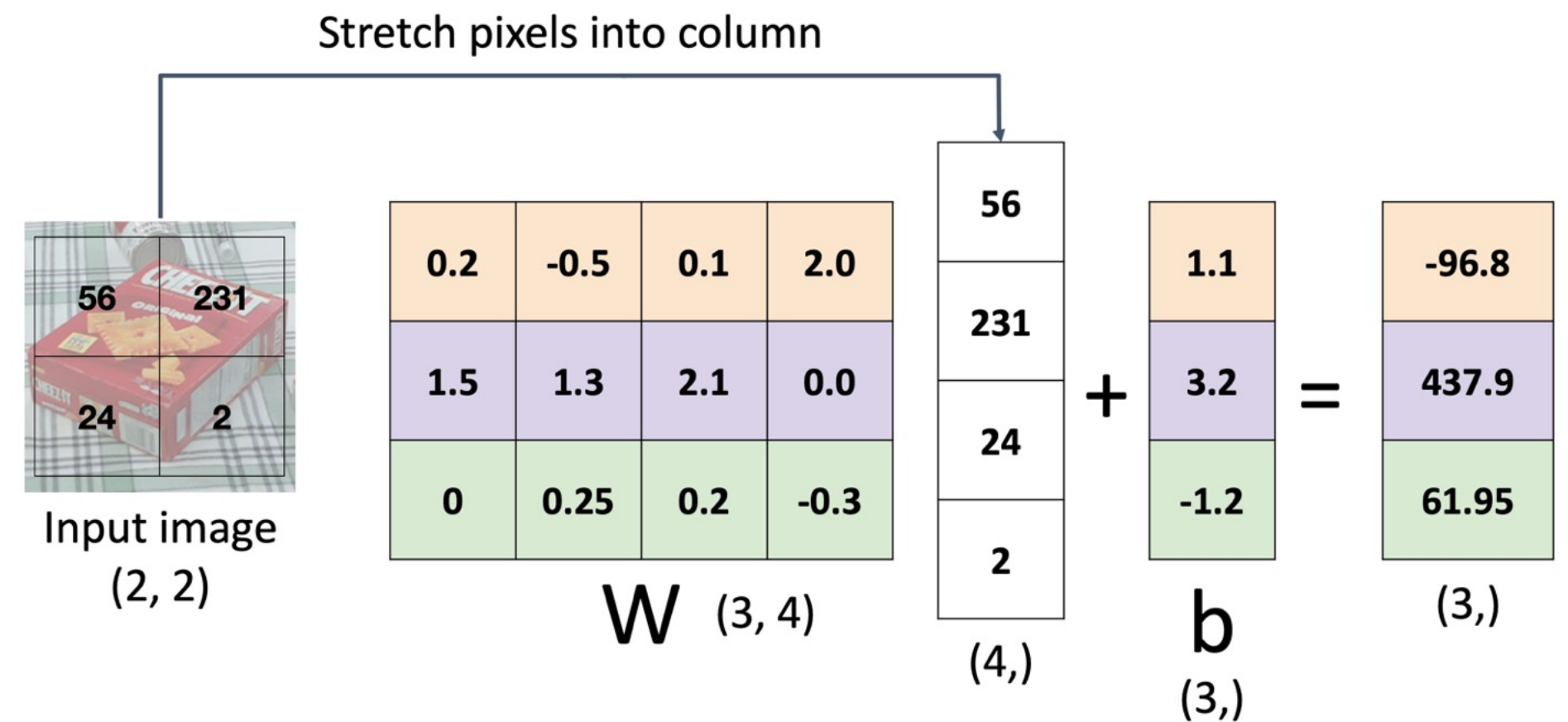
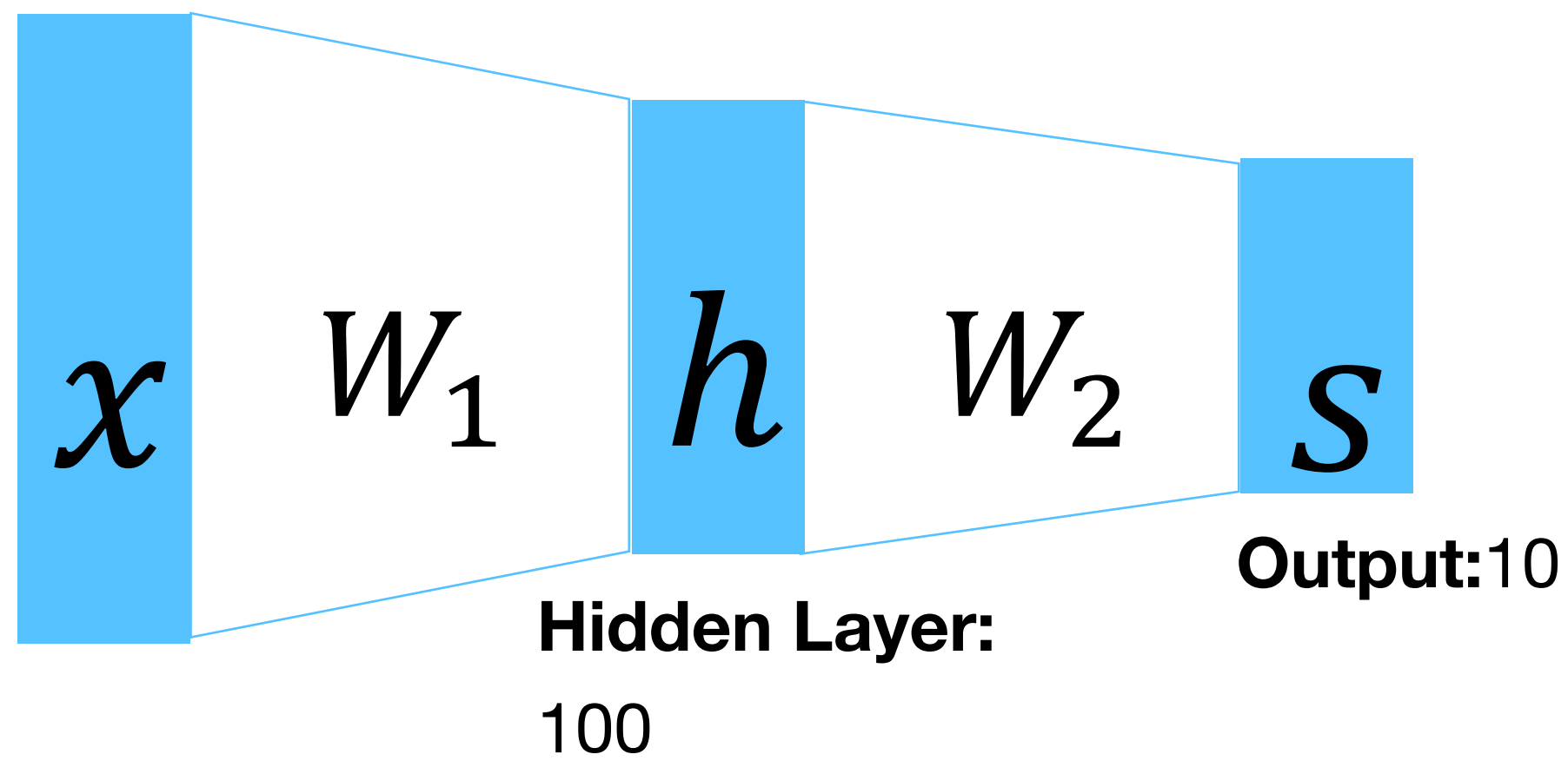


**Problem:** So far our classifiers don't respect the spatial structure of images!

**Solution:** Define new computational nodes that operate on images!

$$f(x) = W_2 \max(0, W_1 x + b_1) + b_2$$

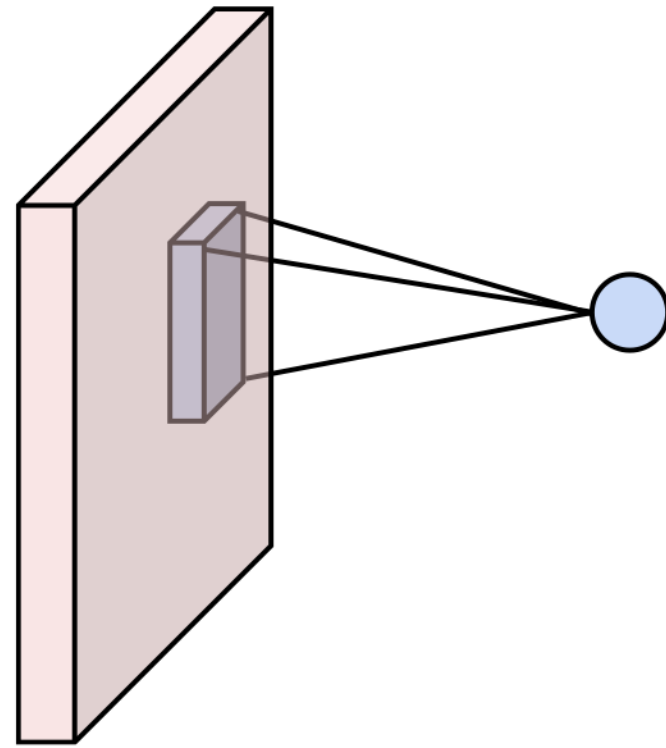
Input:  
3072



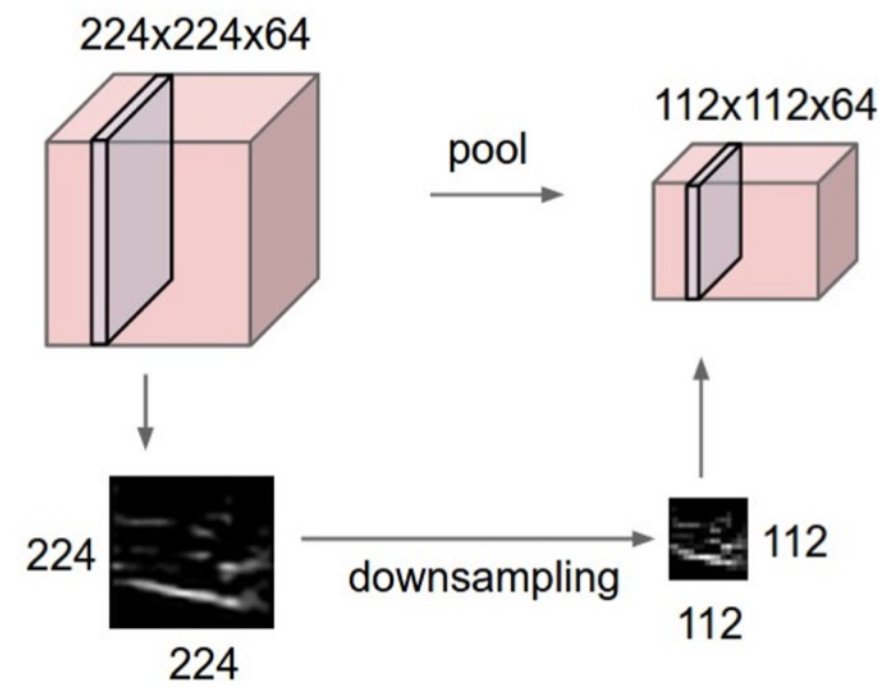


# Components of Convolutional Networks

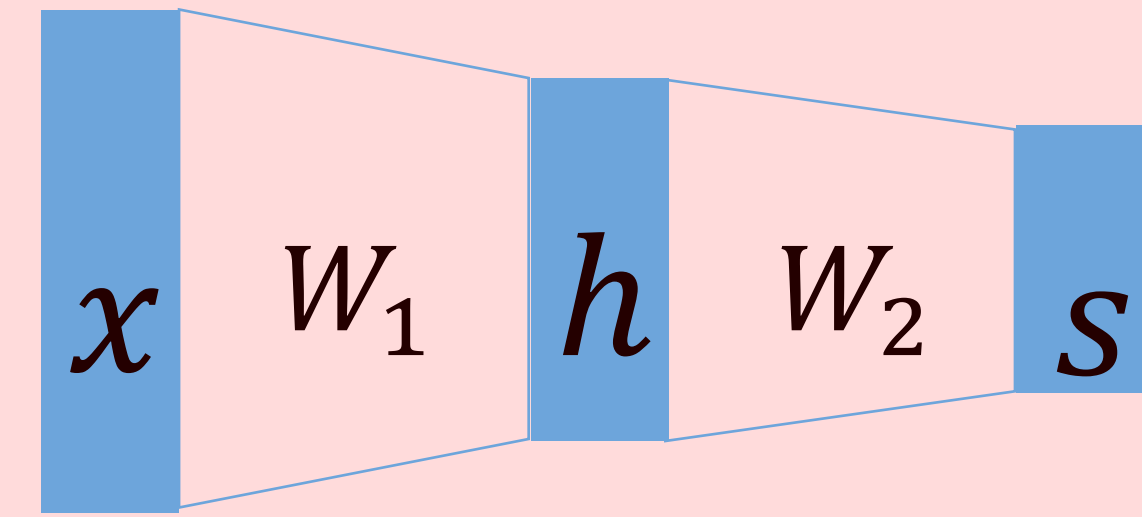
## Convolution Layers



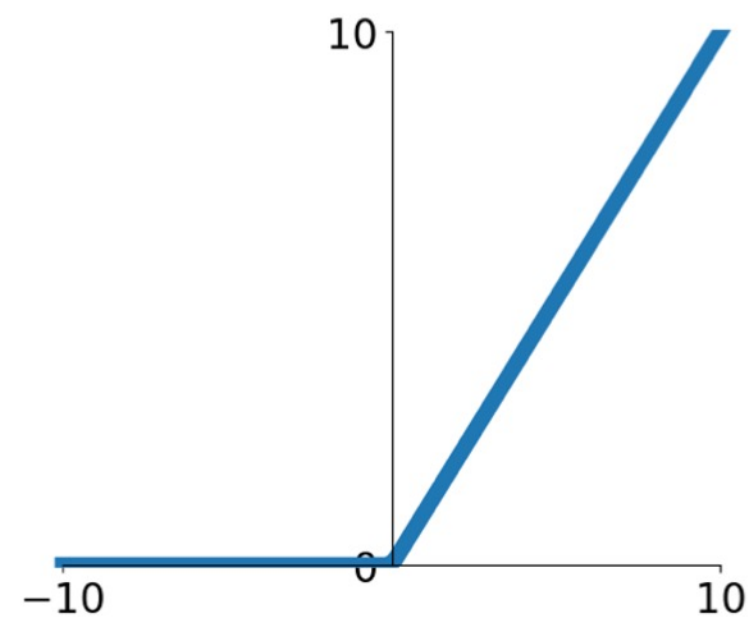
## Pooling Layers



## Fully-Connected Layers



## Activation Function

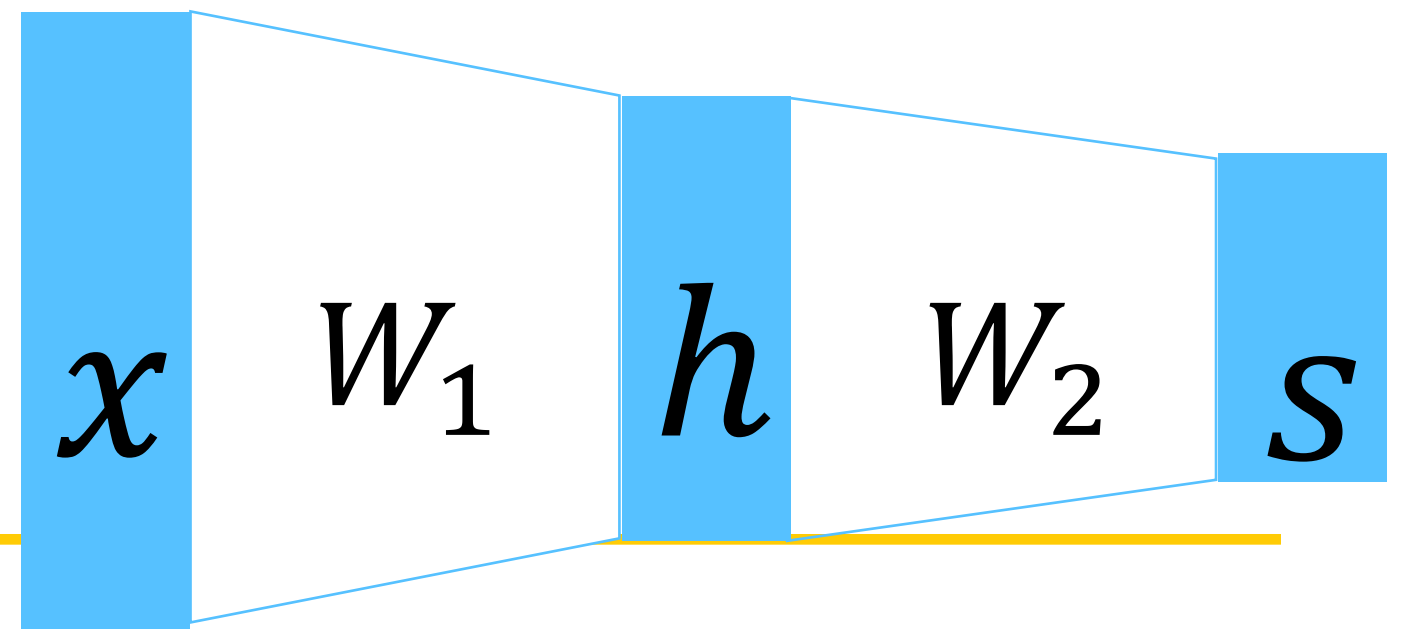


## Normalization

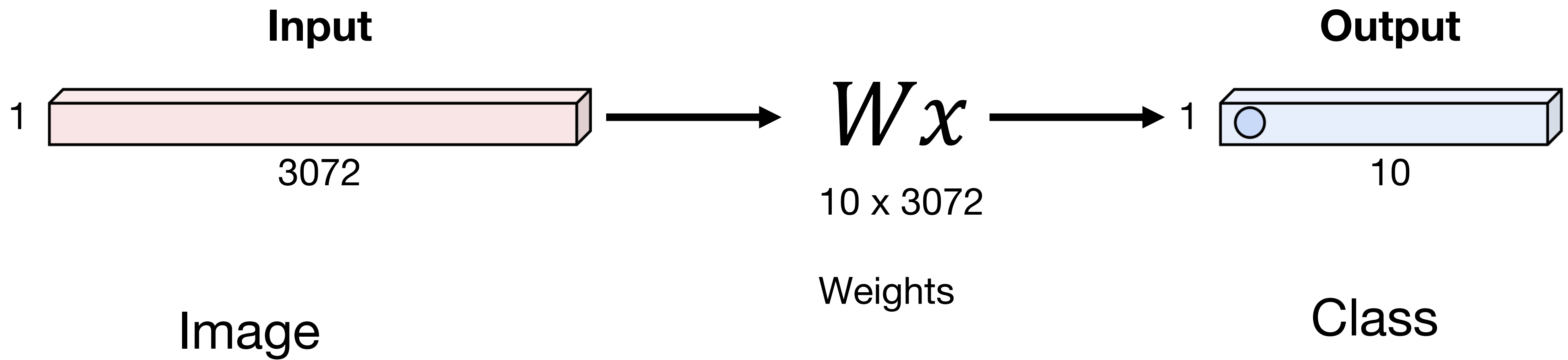
$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \epsilon}}$$



# Fully-Connected Layer



3x32x32 image  $\longrightarrow$  stretch to 3072x1

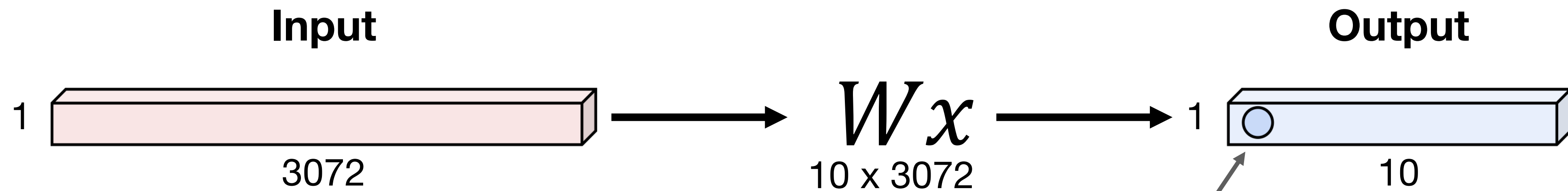






# Fully-Connected Layer

3x32x32 image  $\longrightarrow$  stretch to 3072x1



**1 number:**

The result of taking a dot product between a row of  $W$  and the input



# Fully-Connected Layer

1 number:

3x32x32 image → stretch to 3072x1

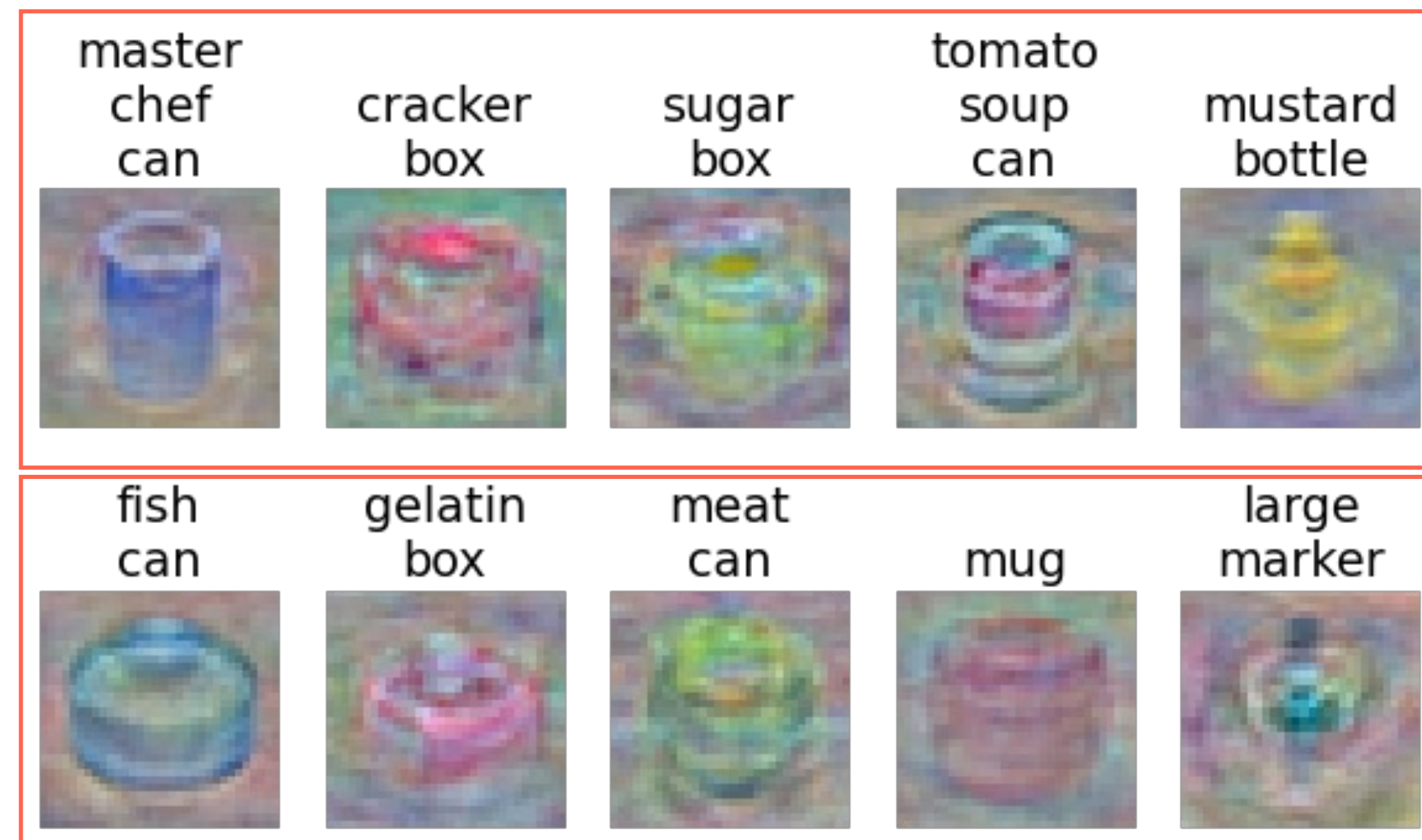
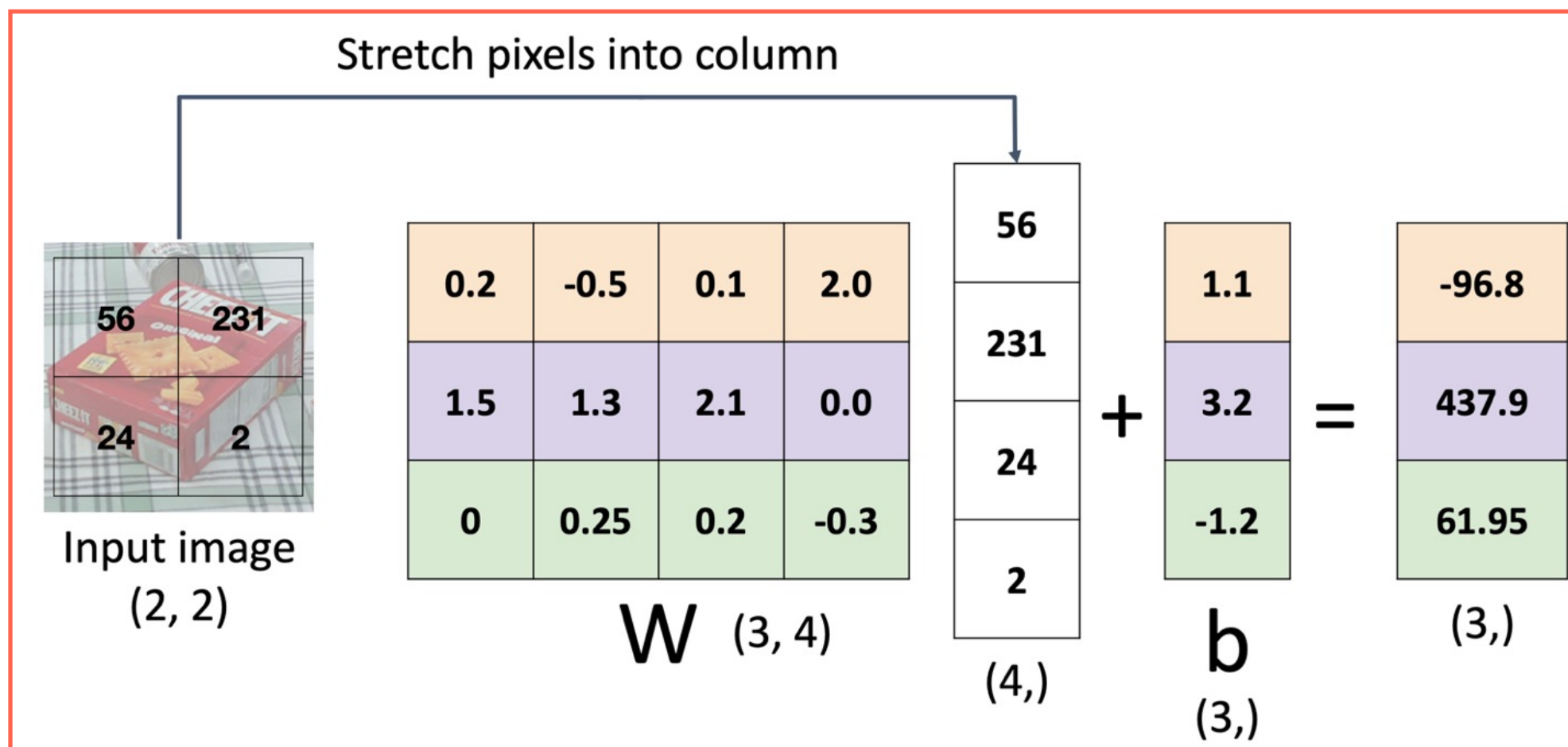
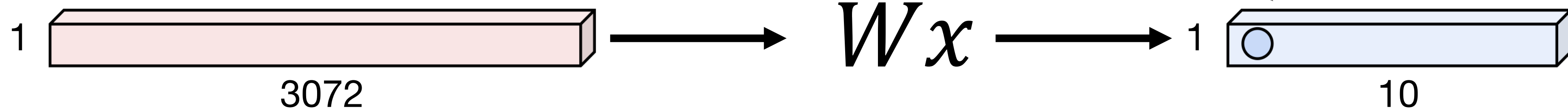
10 x 3072

The result of taking a dot product between a row of  $W$  and the input

Input

Weights

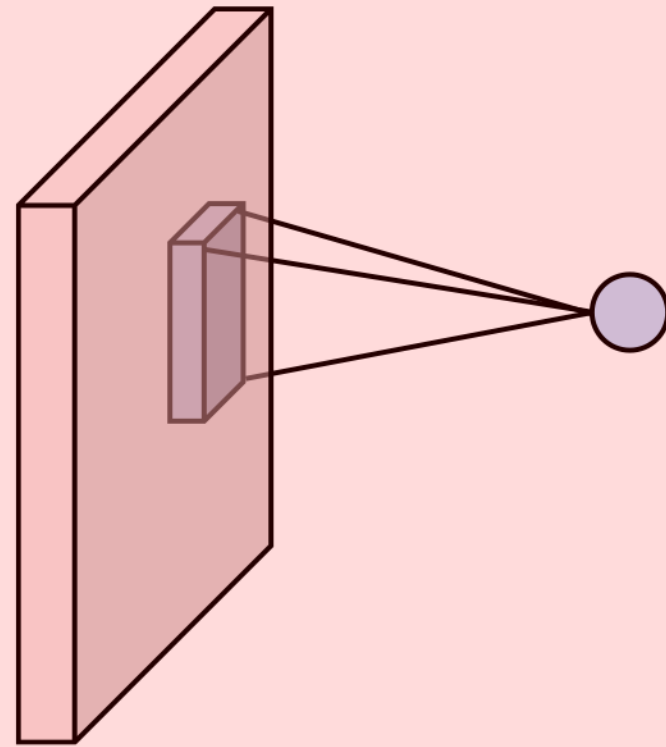
Output



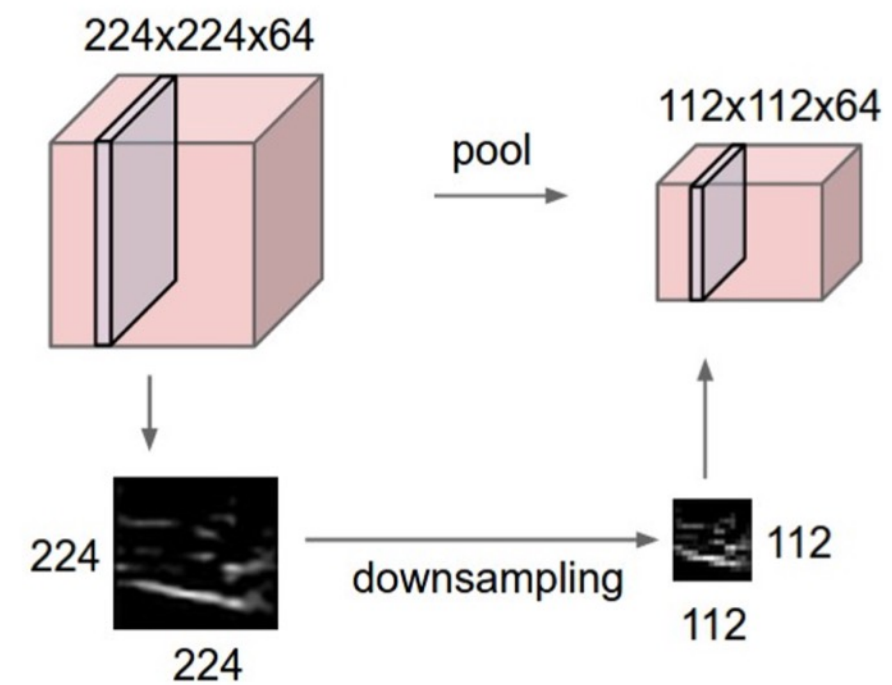


# Components of Convolutional Networks

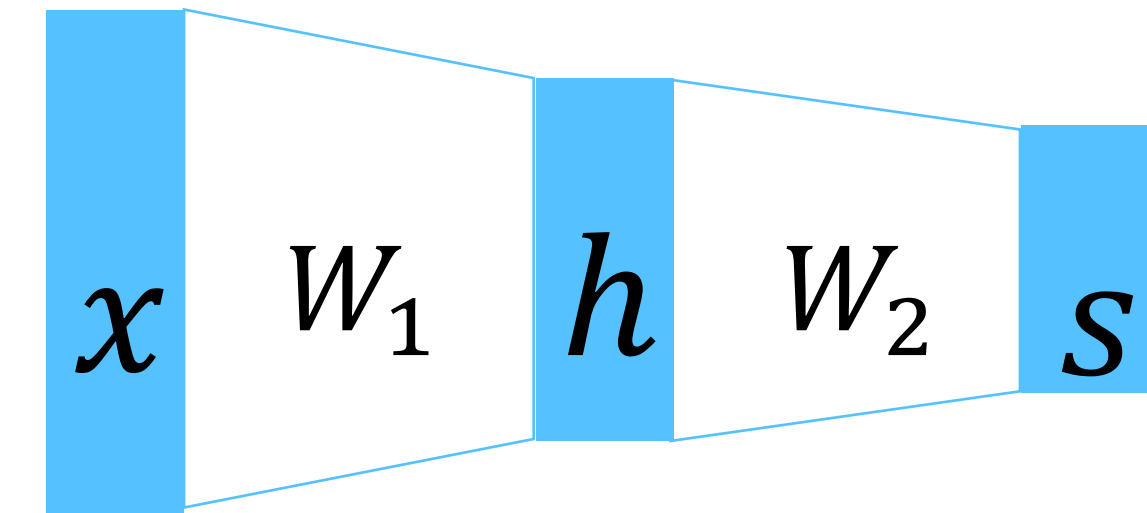
## Convolution Layers



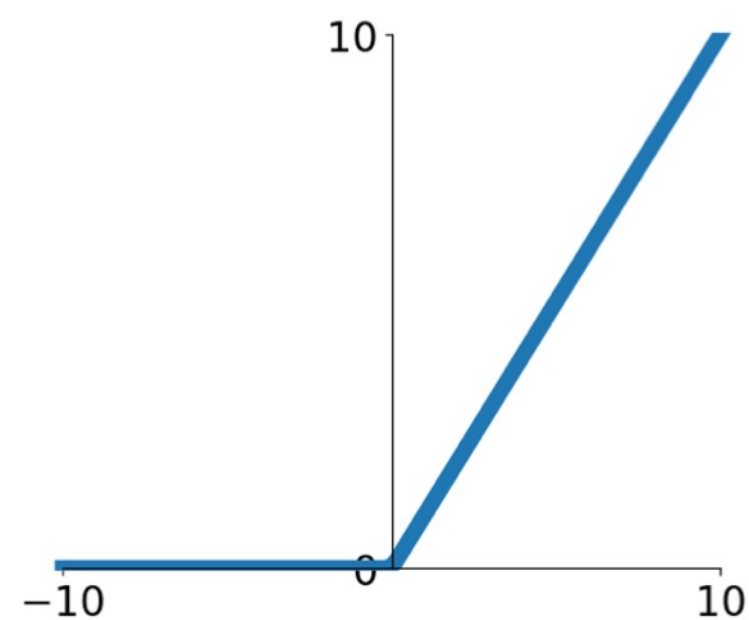
## Pooling Layers



## Fully-Connected Layers



## Activation Function



## Normalization

$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \epsilon}}$$



# Convolution Operation

$3_0$	$3_1$	$2_2$	1	0
$0_2$	$0_2$	$1_0$	3	1
$3_0$	$1_1$	$2_2$	2	3
2	0	0	2	2
2	0	0	0	1

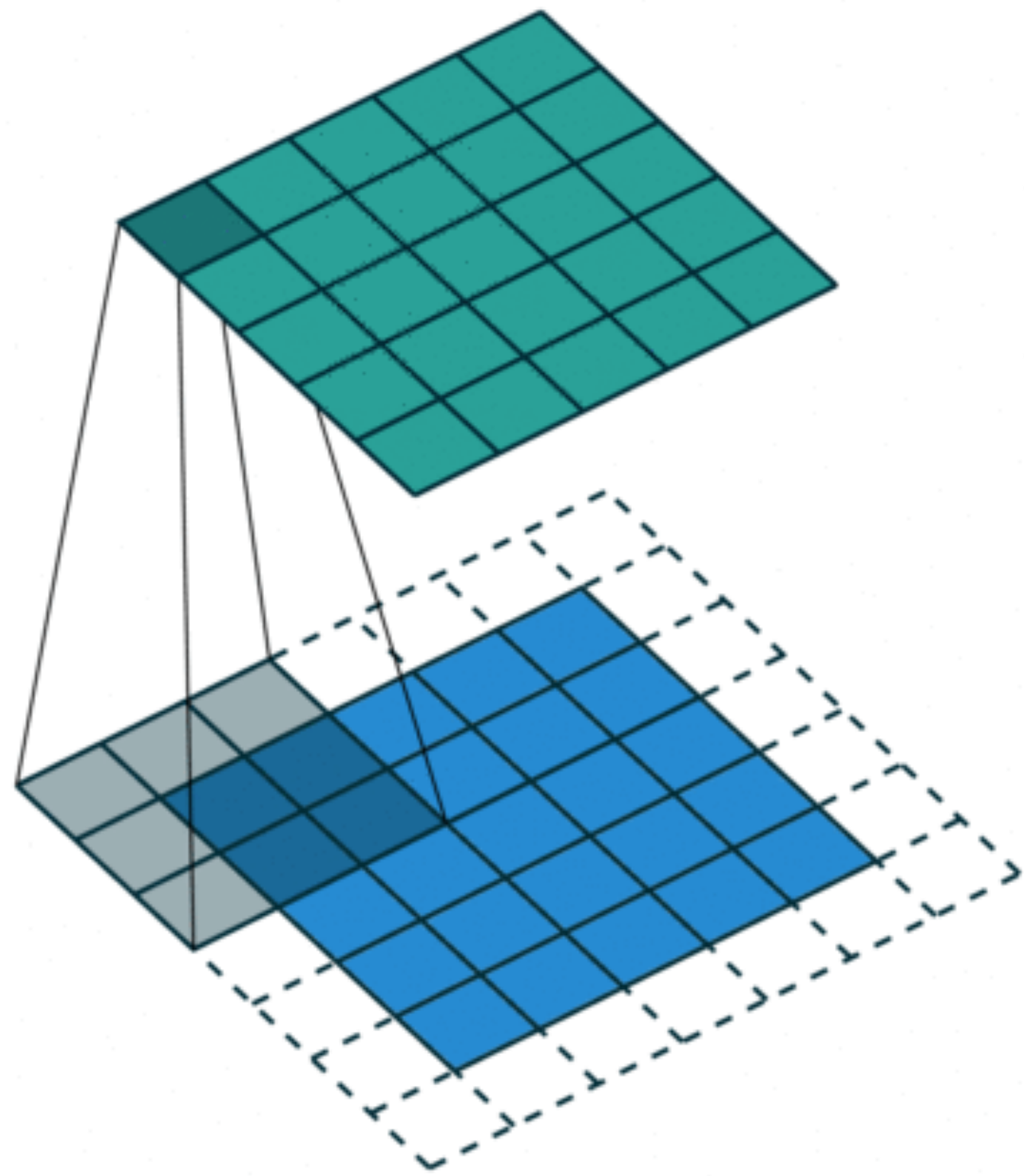
12.0	12.0	17.0
10.0	17.0	19.0
9.0	6.0	14.0

Kernel:  
0 1 2  
2 2 0  
0 1 2  
(3x3)

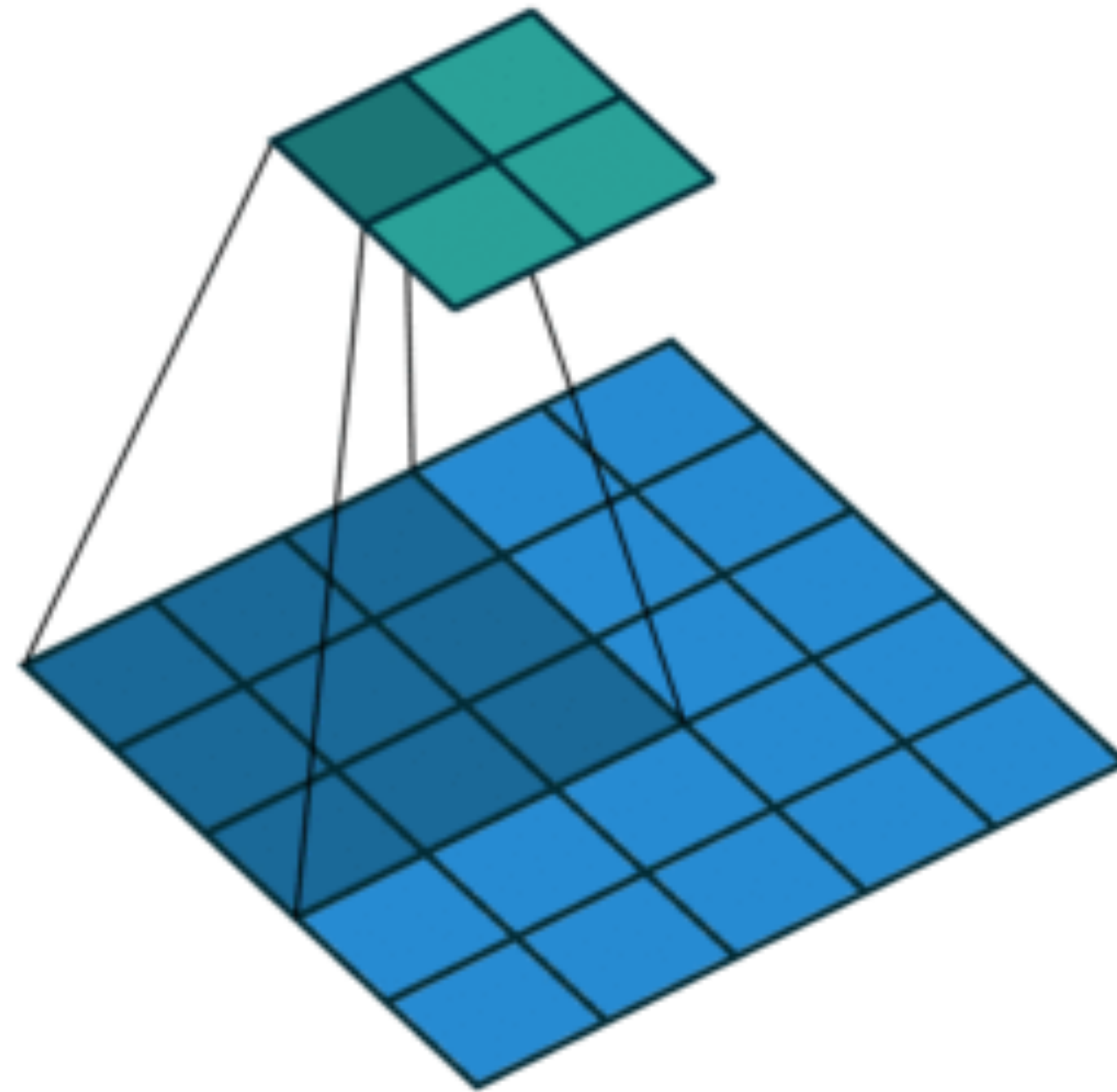
<https://towardsdatascience.com/intuitively-understanding-convolutions-for-deep-learning-1f6f42faee1>



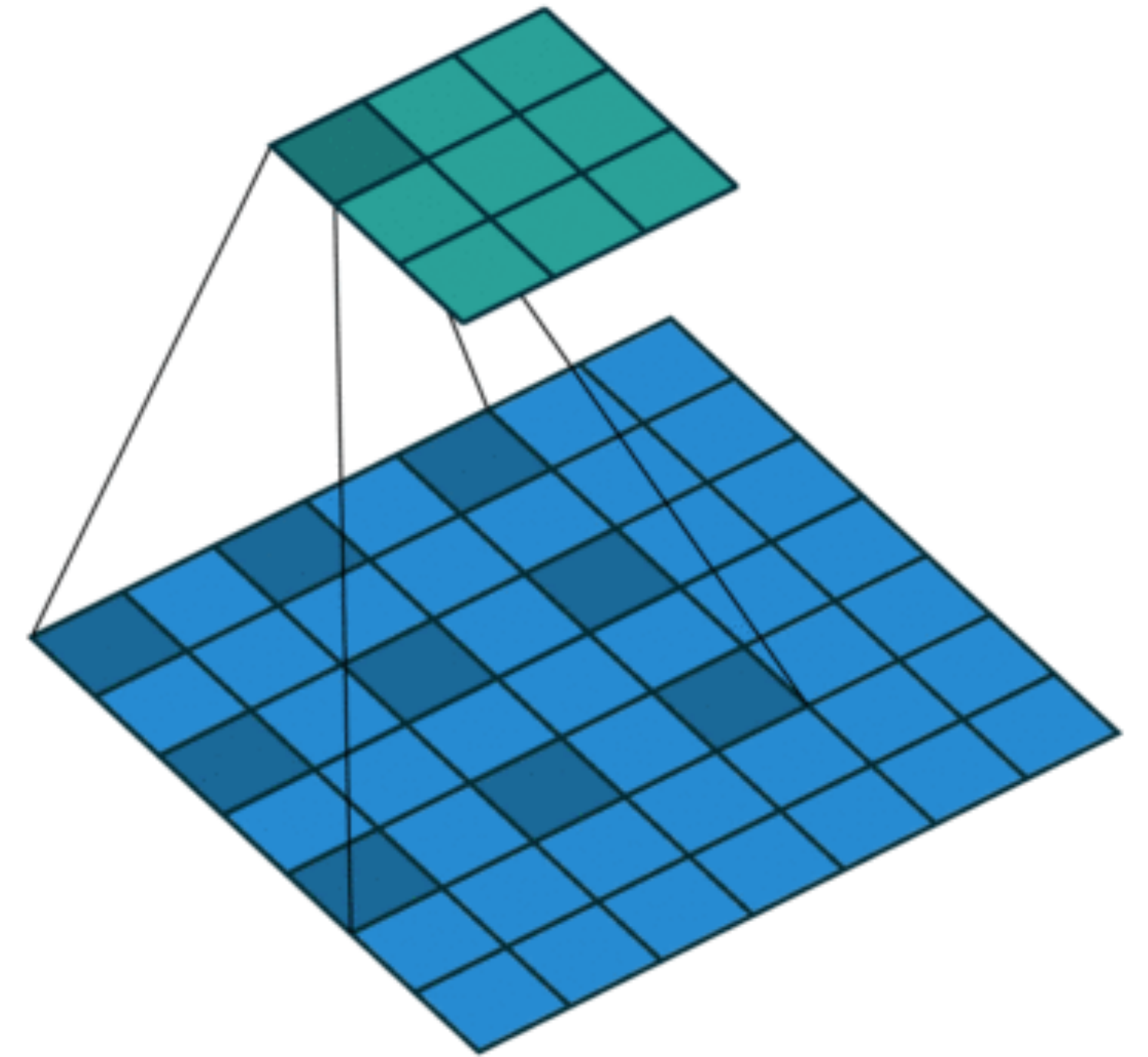
# Convolution Operation



Padding



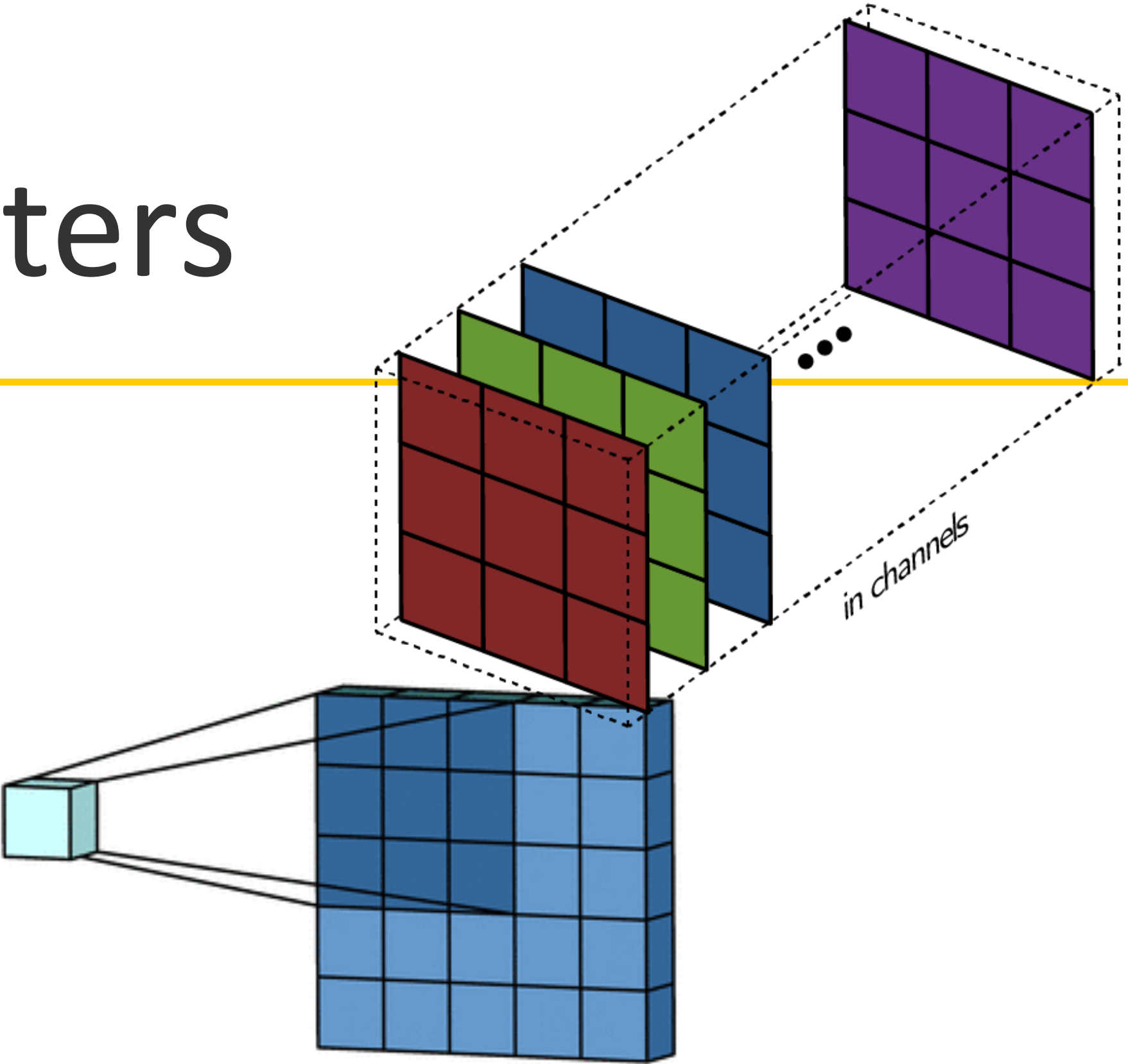
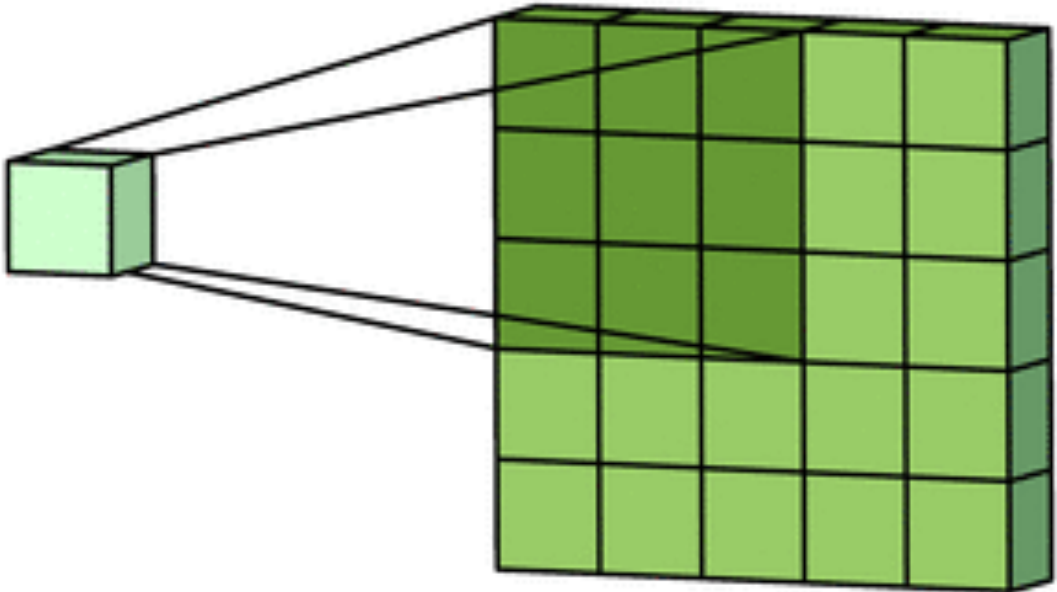
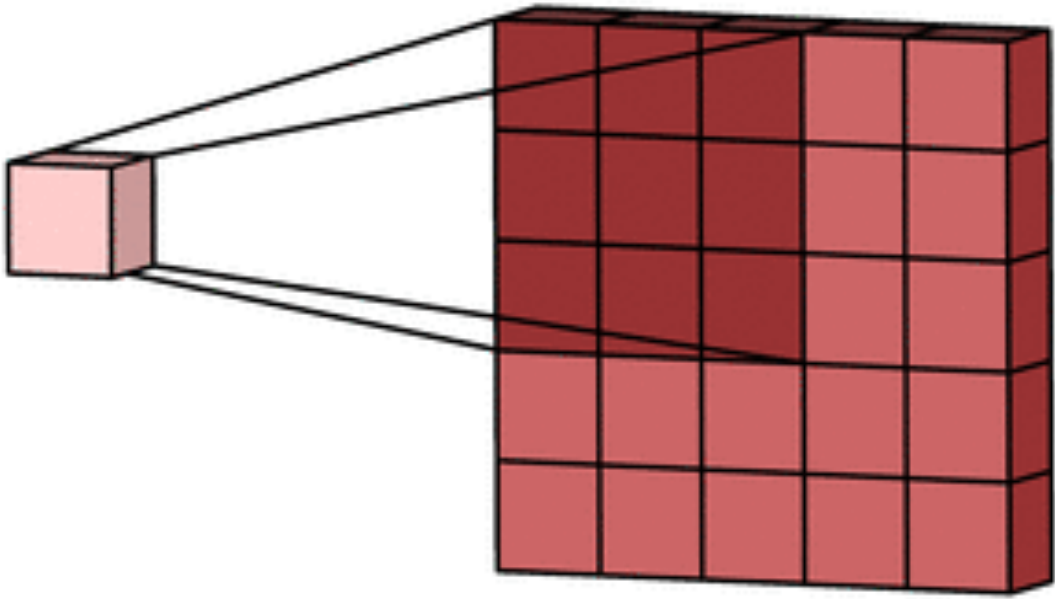
Stride = 2



dilation = 2

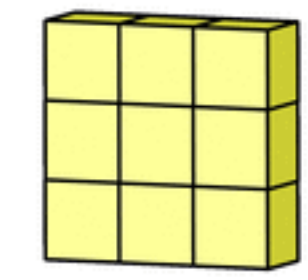
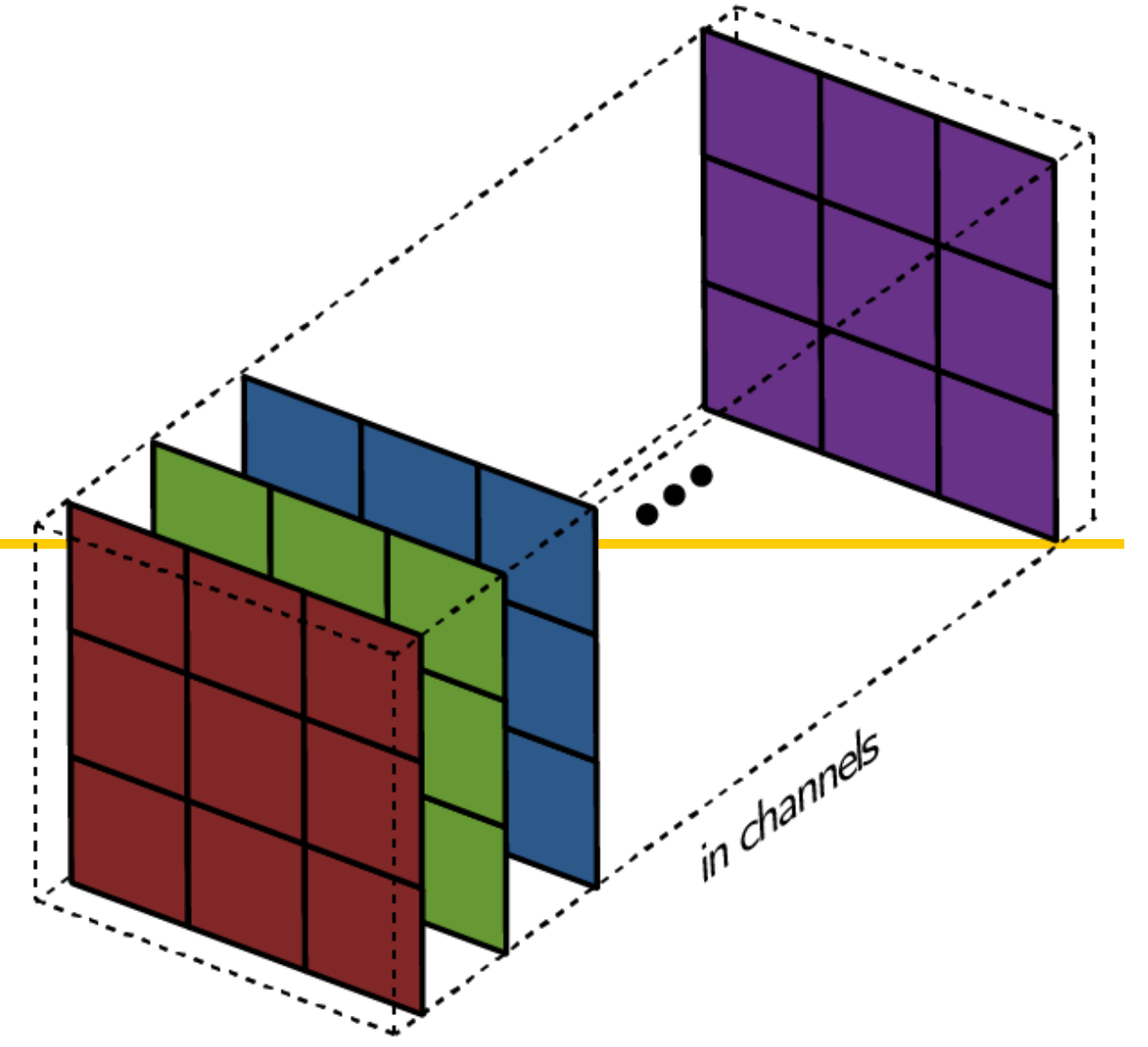
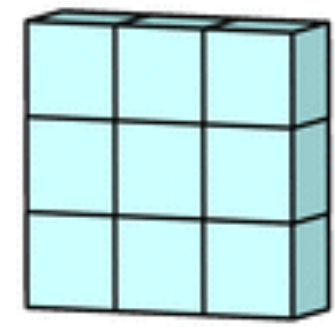
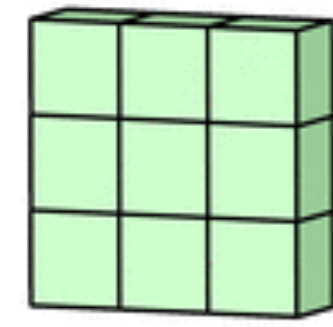
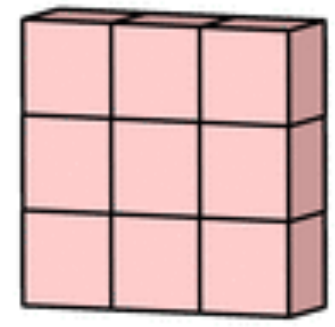


# Convolution Filters





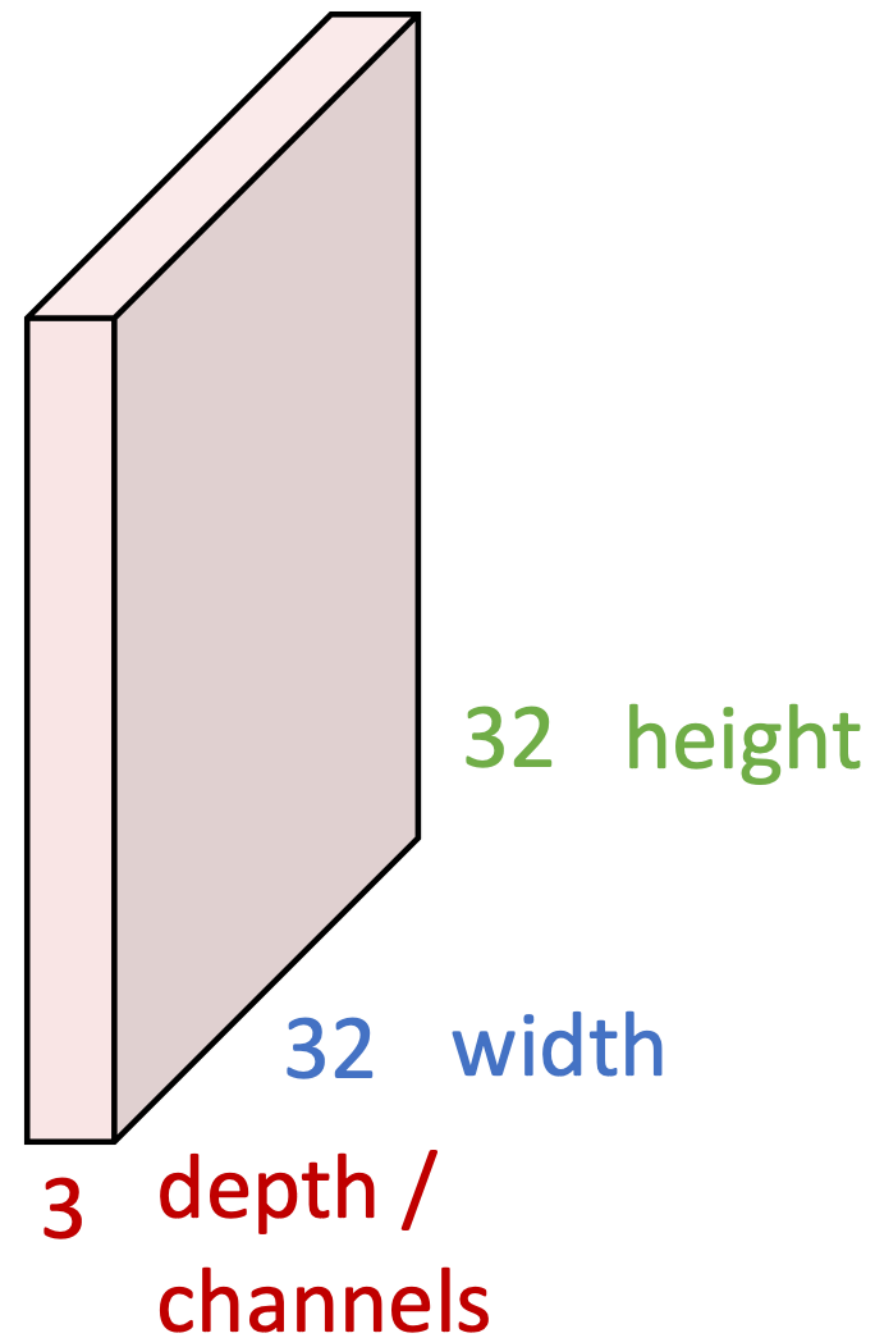
# Convolution Filters





# Convolution Layer

3x32x32 image: preserve spatial structure



3x5x5 filter



**Convolve** the filter with the image i.e., “slide over the image spatially, computing dot products”





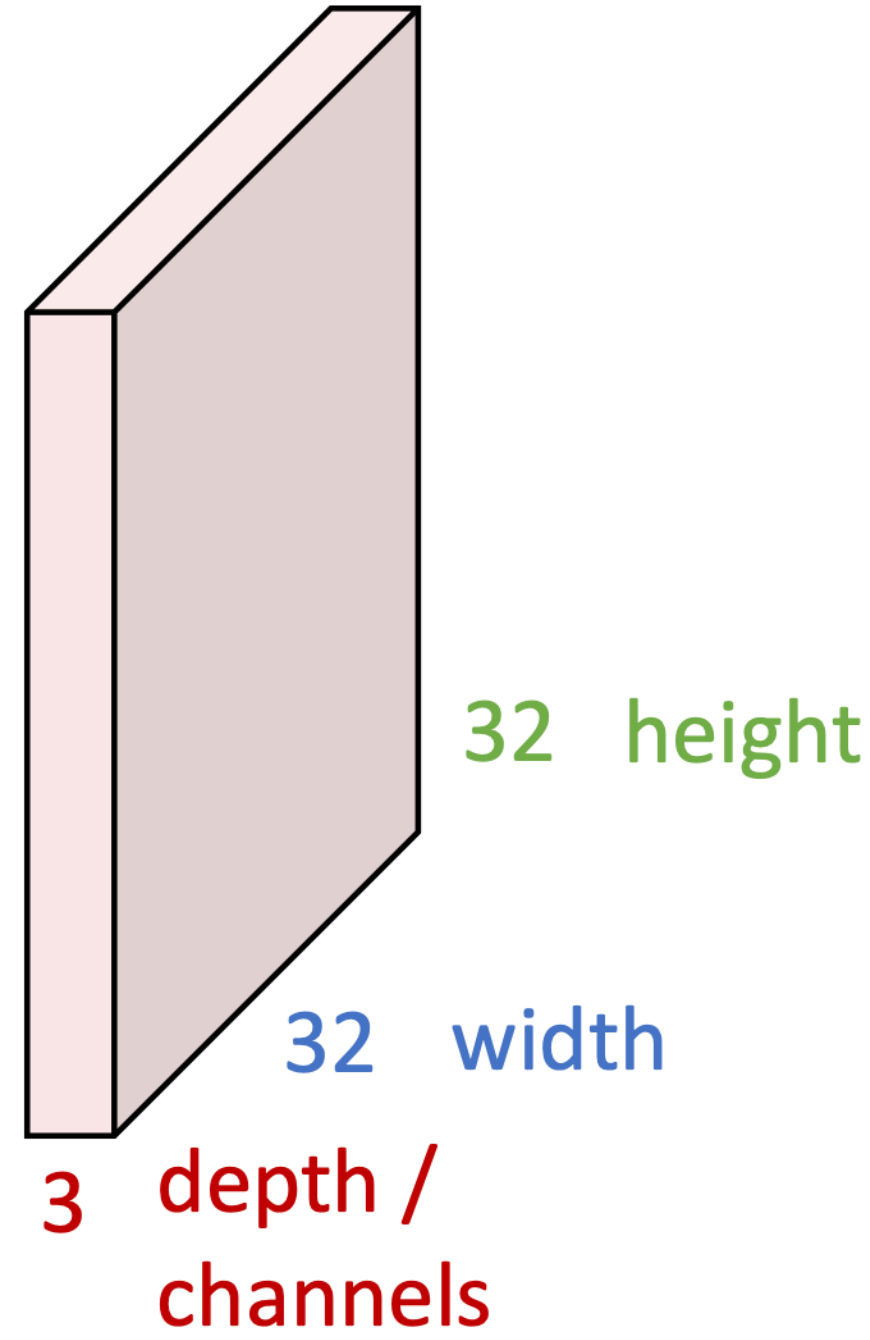
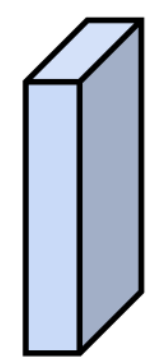
# Convolution Layer

3x32x32 image



Filters always extend the full depth of the input volume

3x5x5 filter



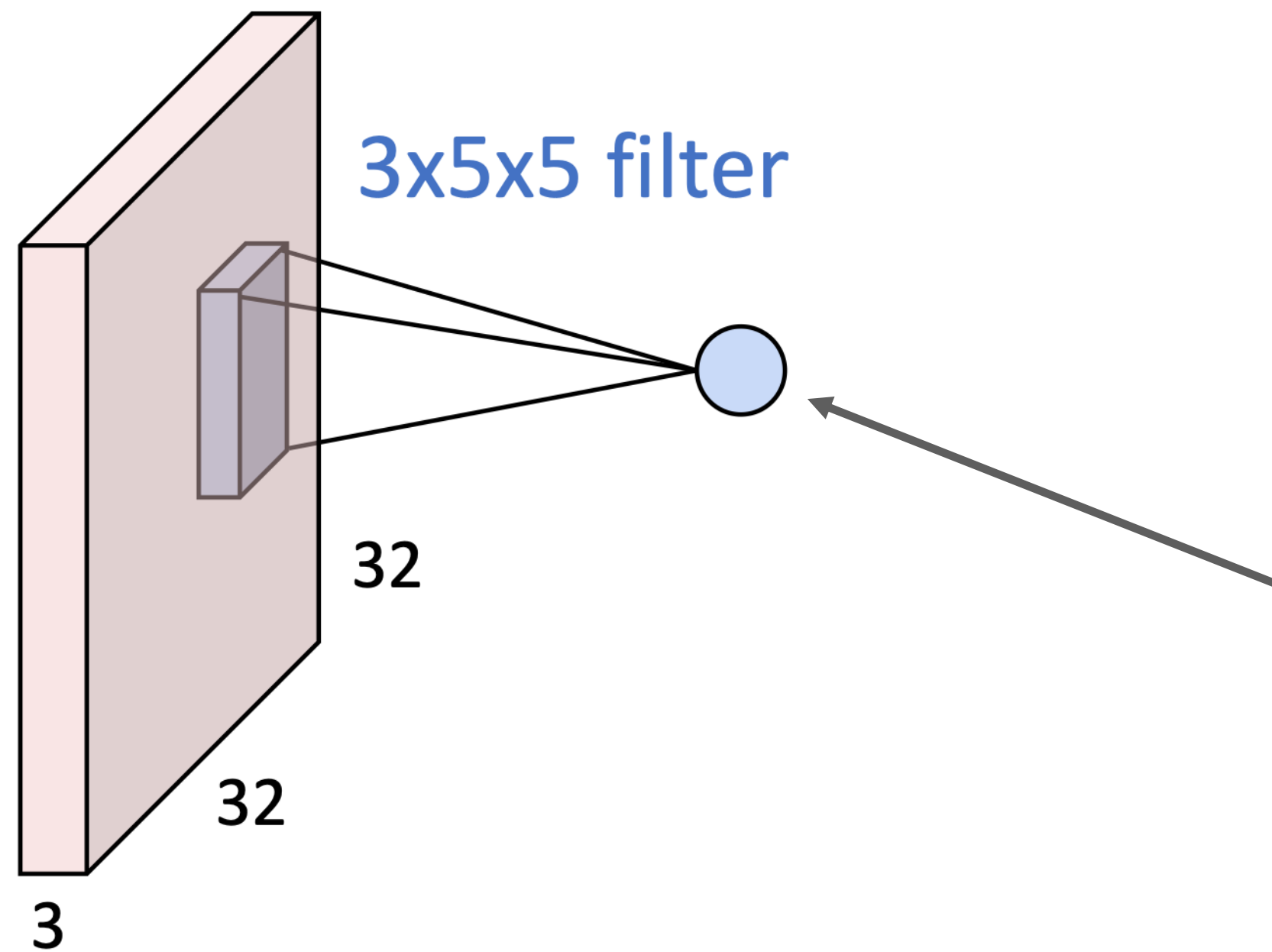
**Convolve** the filter with the image i.e. “slide over the image spatially, computing dot products”



# Convolution Layer

3x32x32 image

$$L_{out} = (L_{in} + 2 * padding - dilation * (kernel - 1) - 1) / stride + 1$$



**1 number:**

The result of taking a dot product between the filter and a small 3x5x5 portion of the image

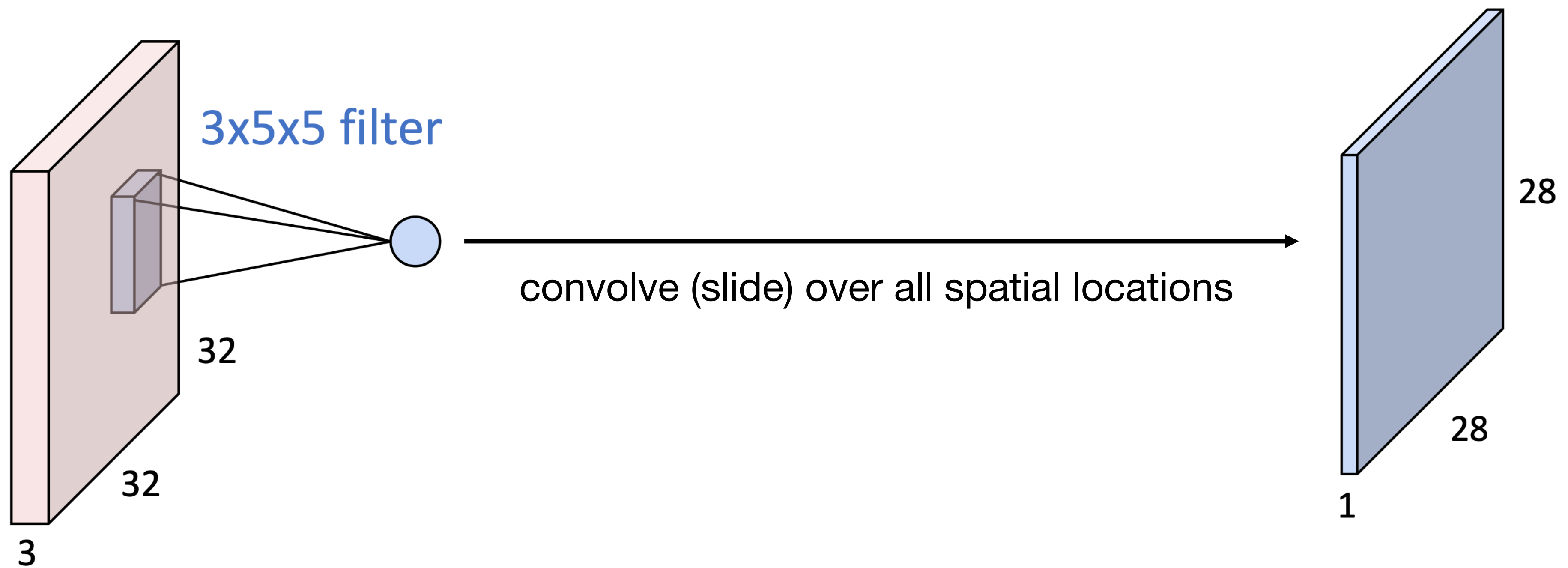
(i.e.  $3*5*5=75$ -dimensional dot product + bias)  
 $w^T x + b$



# Convolution Layer

3x32x32 image

1x28x28 activation map

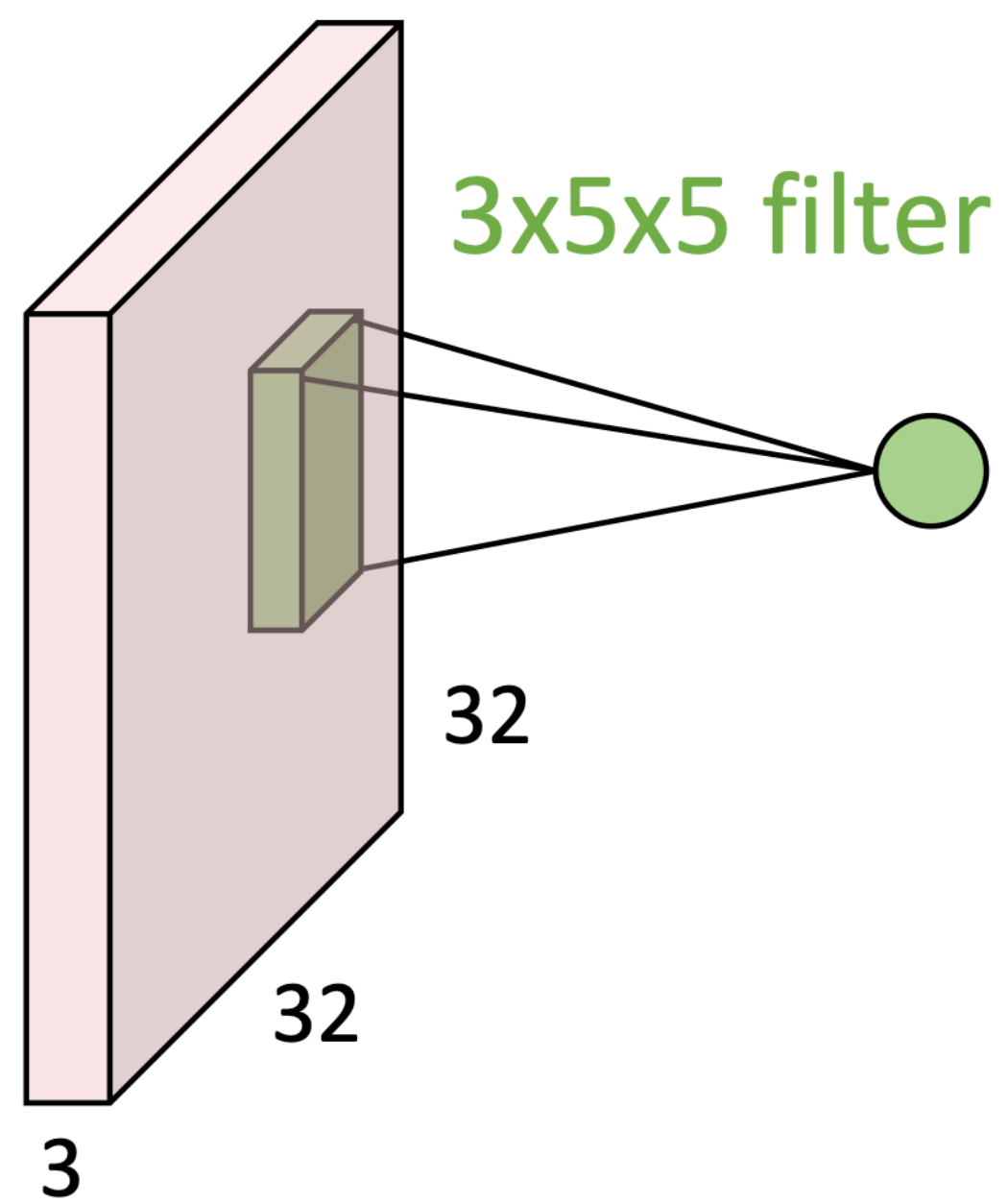




# Convolution Layer

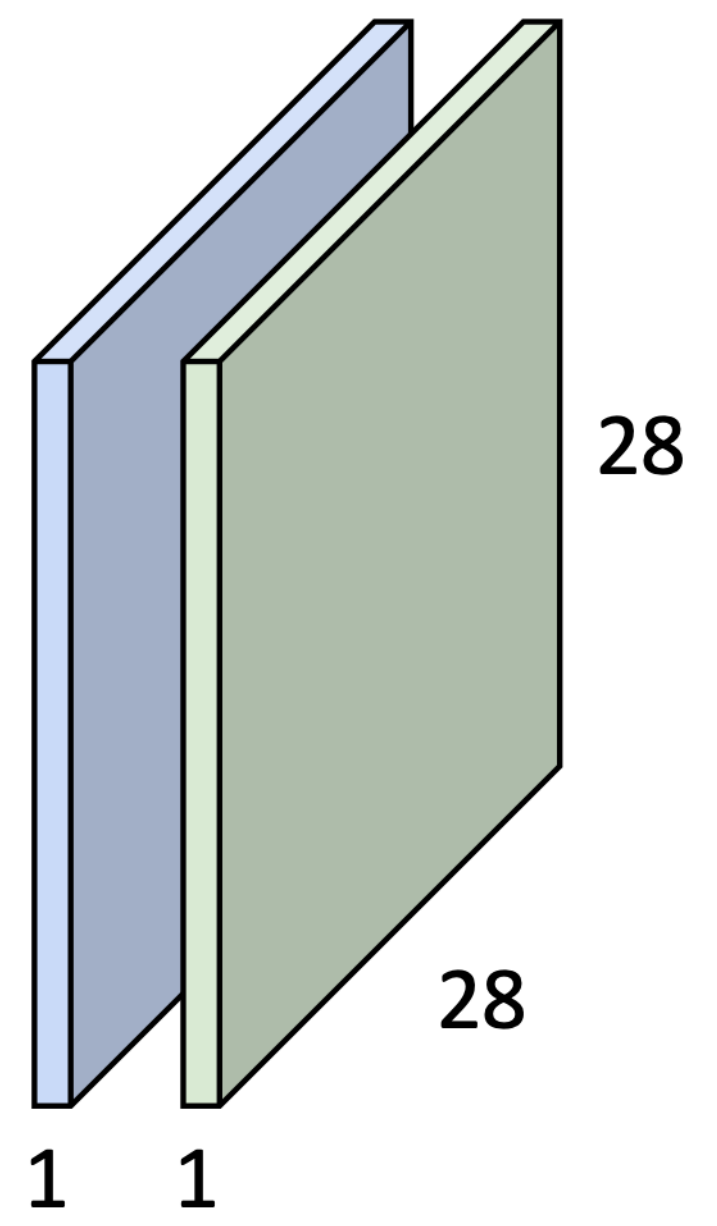
3x32x32 image

two 1x28x28 activation map



Consider repeating with a second (green) filter

convolve (slide) over all spatial locations

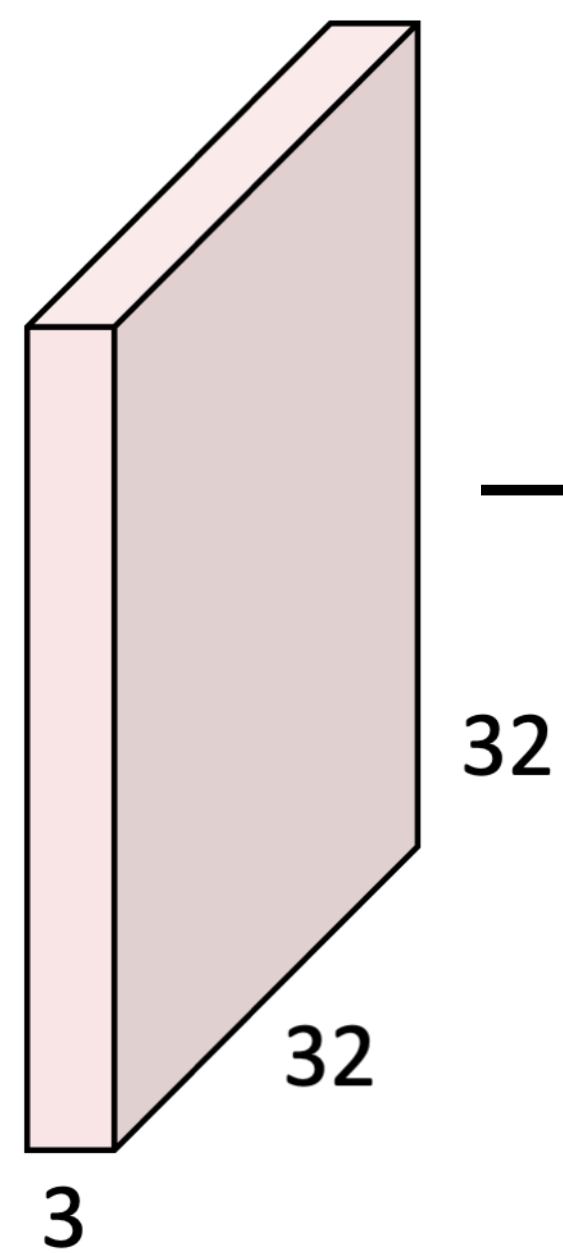




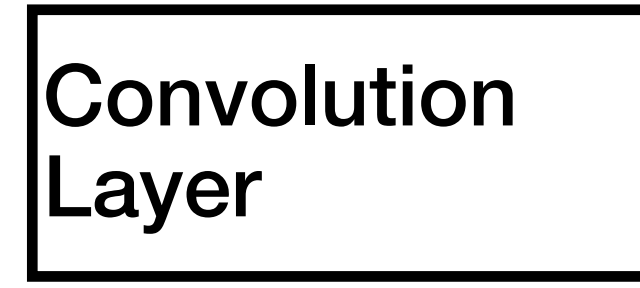
# Convolution Layer

3x32x32 image

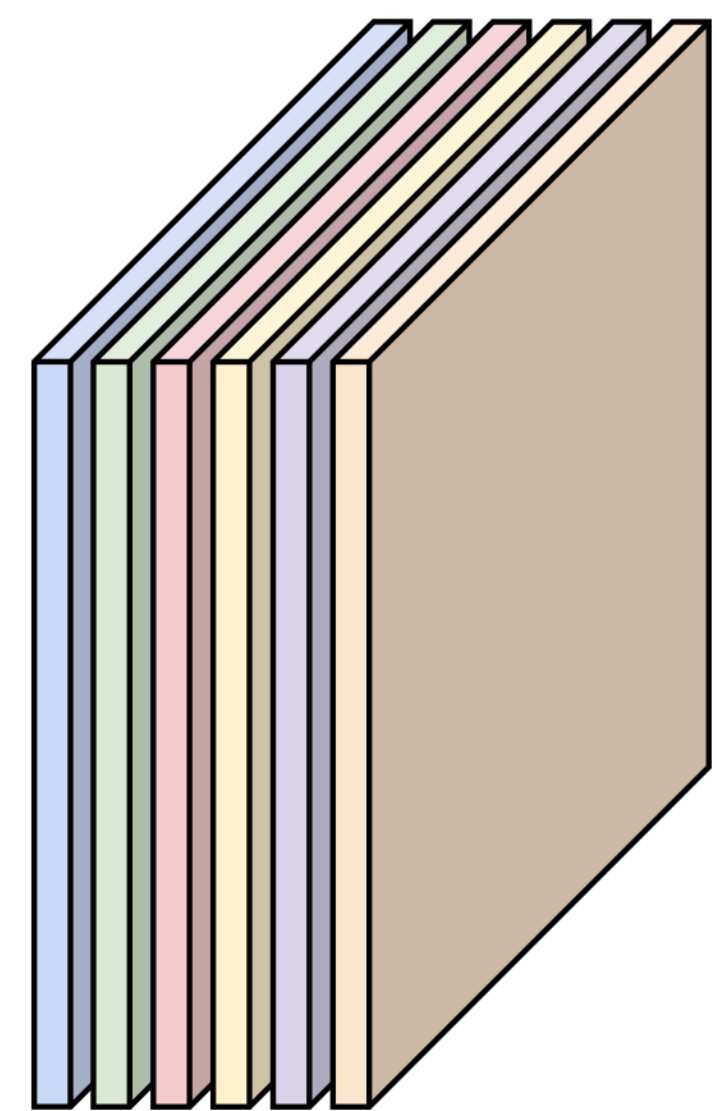
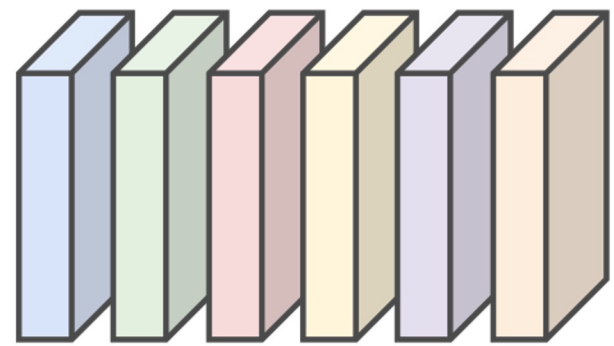
six 1x28x28 activation map



Consider 6 filters,  
each 3x5x5



6x3x5x5 filters



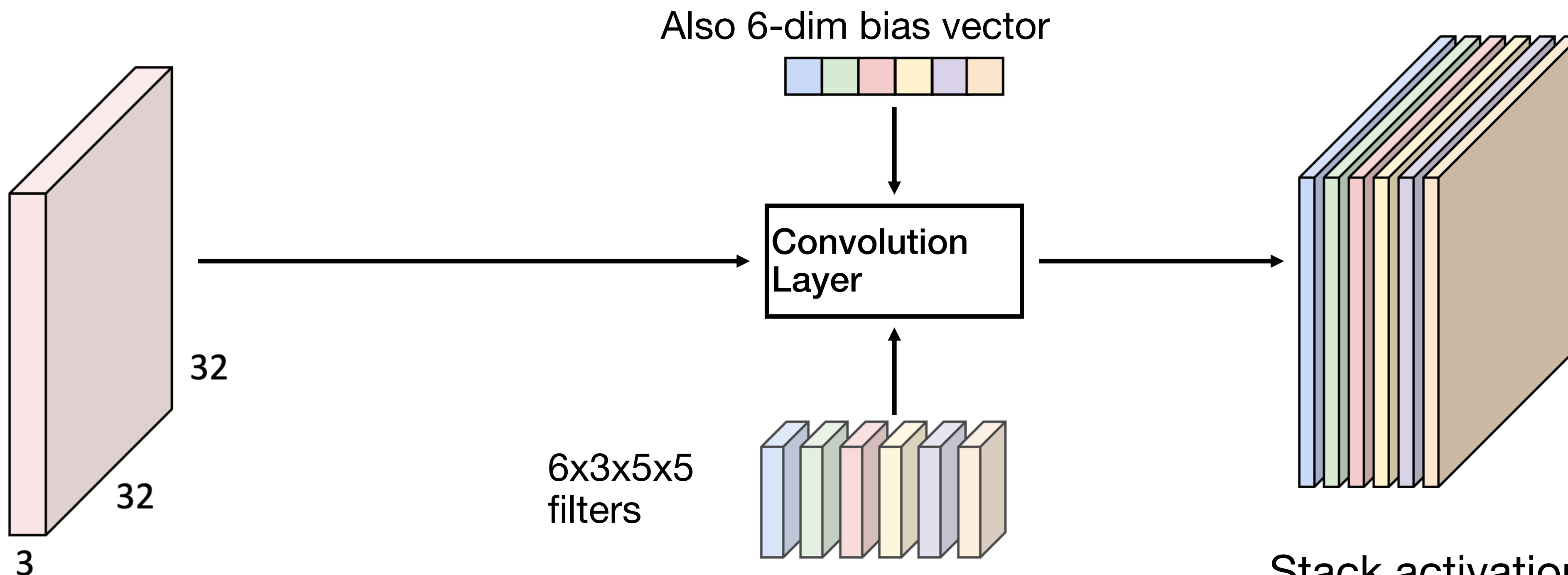
Stack activations to get  
a 6x28x28 output image



# Convolution Layer

3x32x32 image

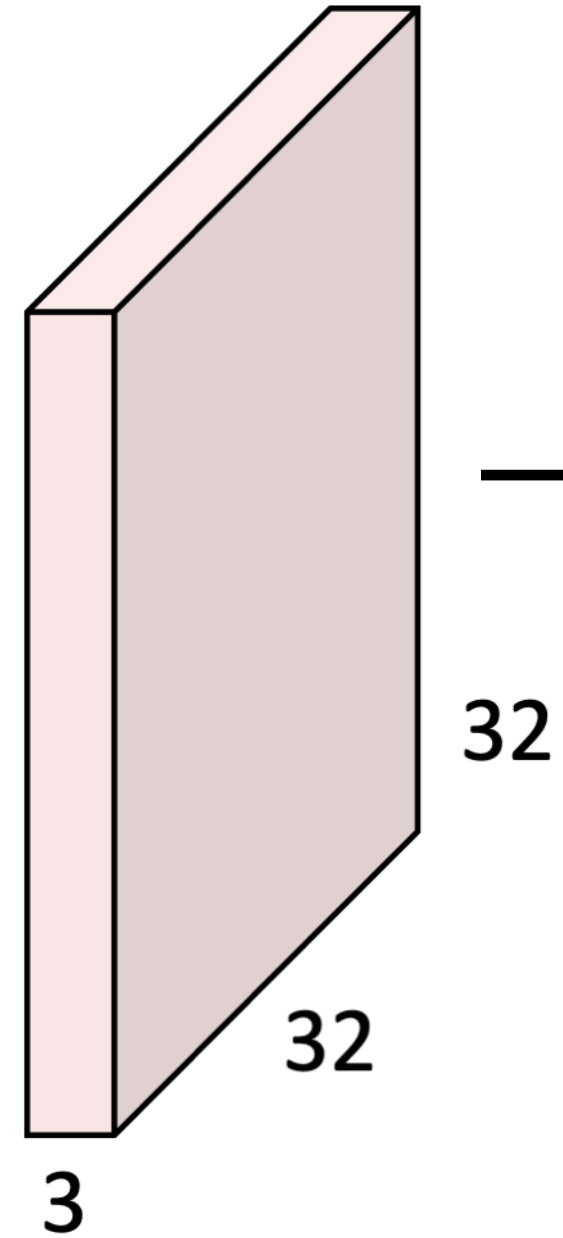
six 1x28x28 activation map



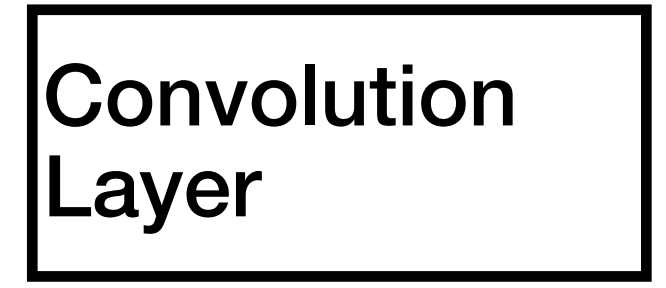


# Convolution Layer

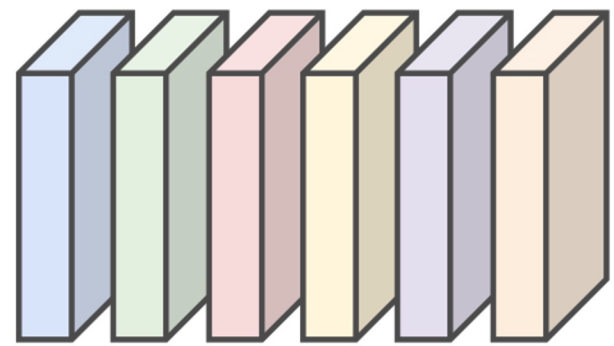
3x32x32 image



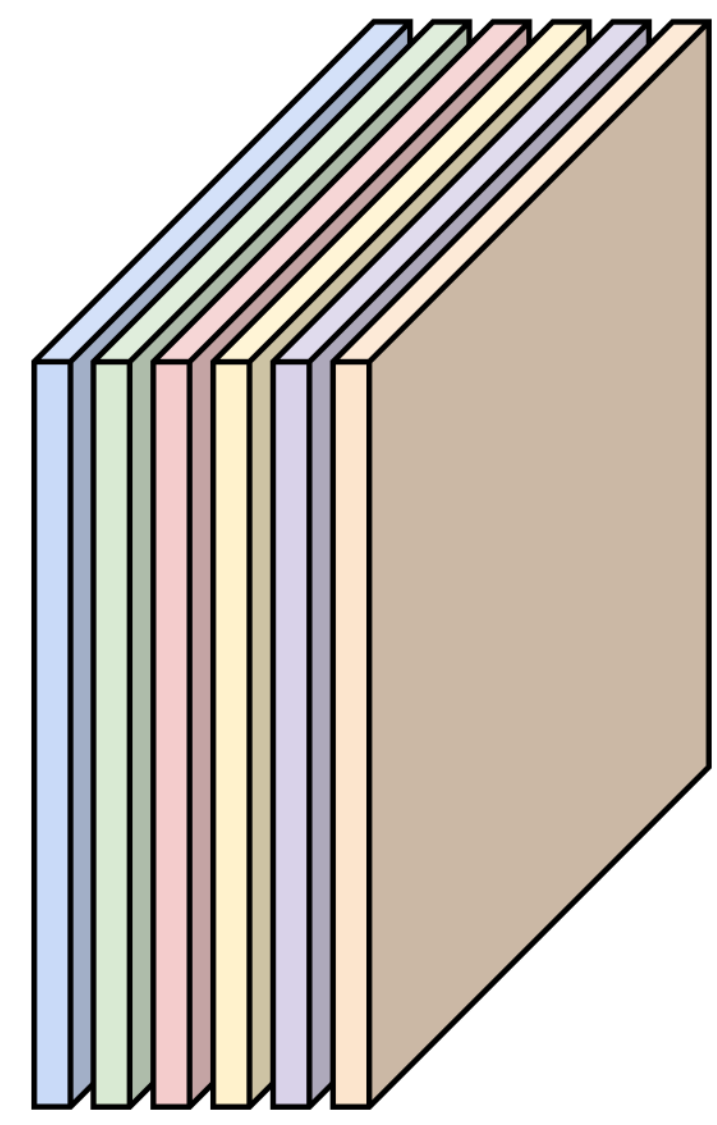
Also 6-dim bias vector



6x3x5x5 filters



28x28 grid, at each point a 6-dim vector



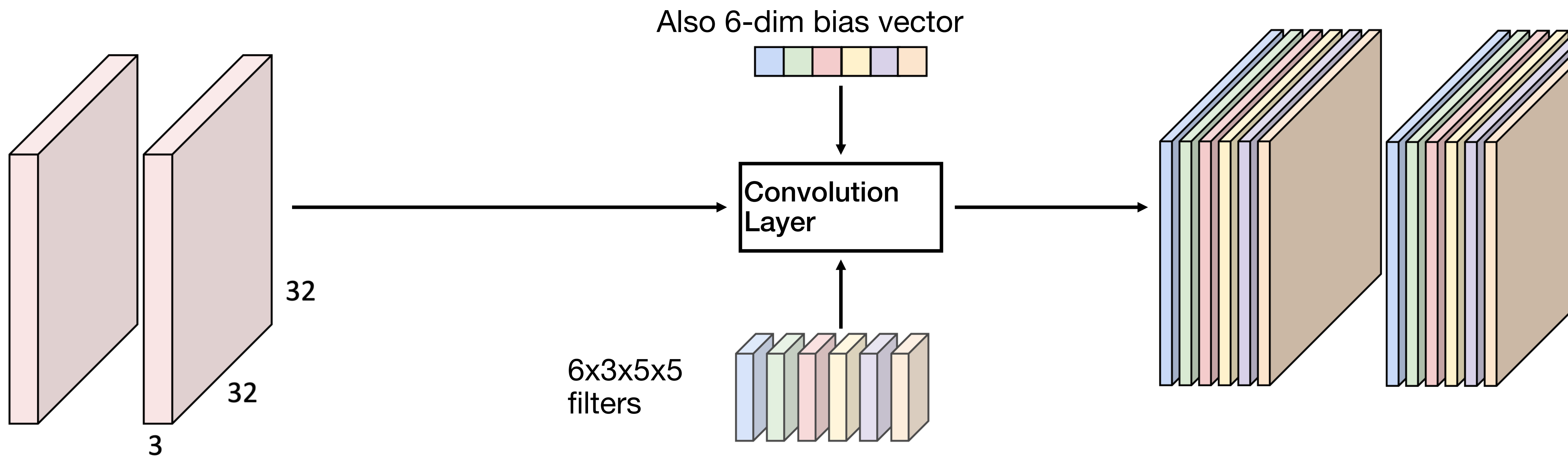
Stack activations to get a 6x28x28 output image



# Convolution Layer

2x3x32x32  
batch of images

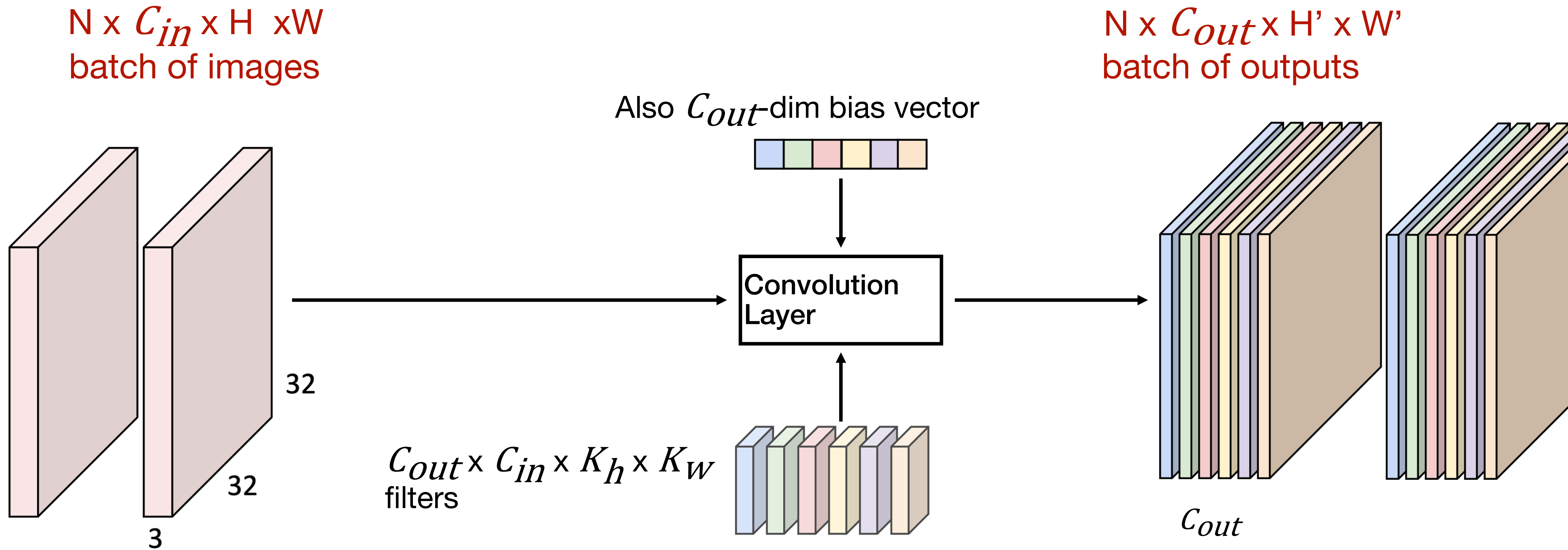
2x6x28x28  
batch of outputs





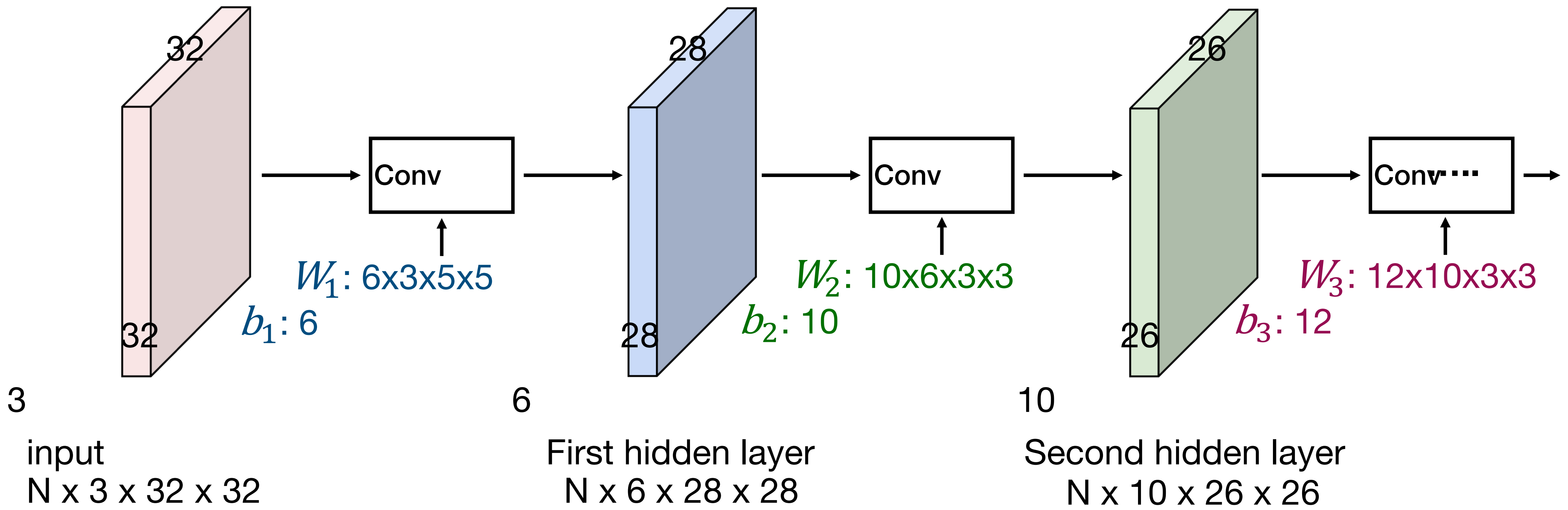


# Convolution Layer: General dimensions





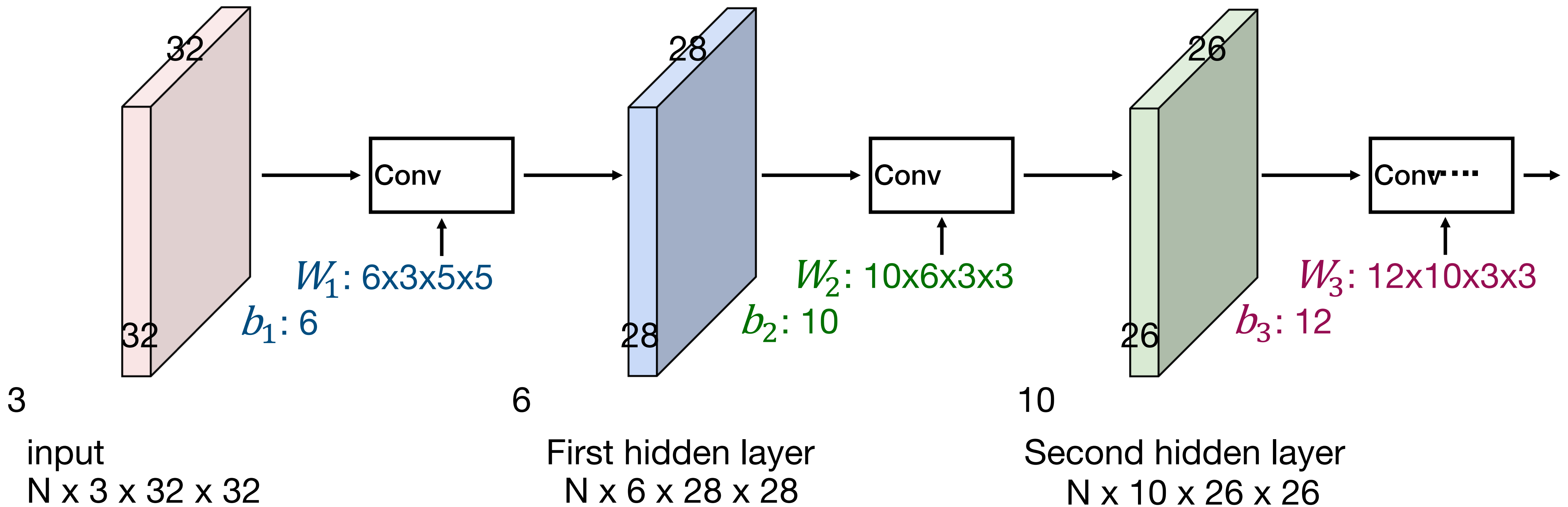
# Stacking Convolutions





# Stacking Convolutions

Q: What happens if we stack two convolution layers?

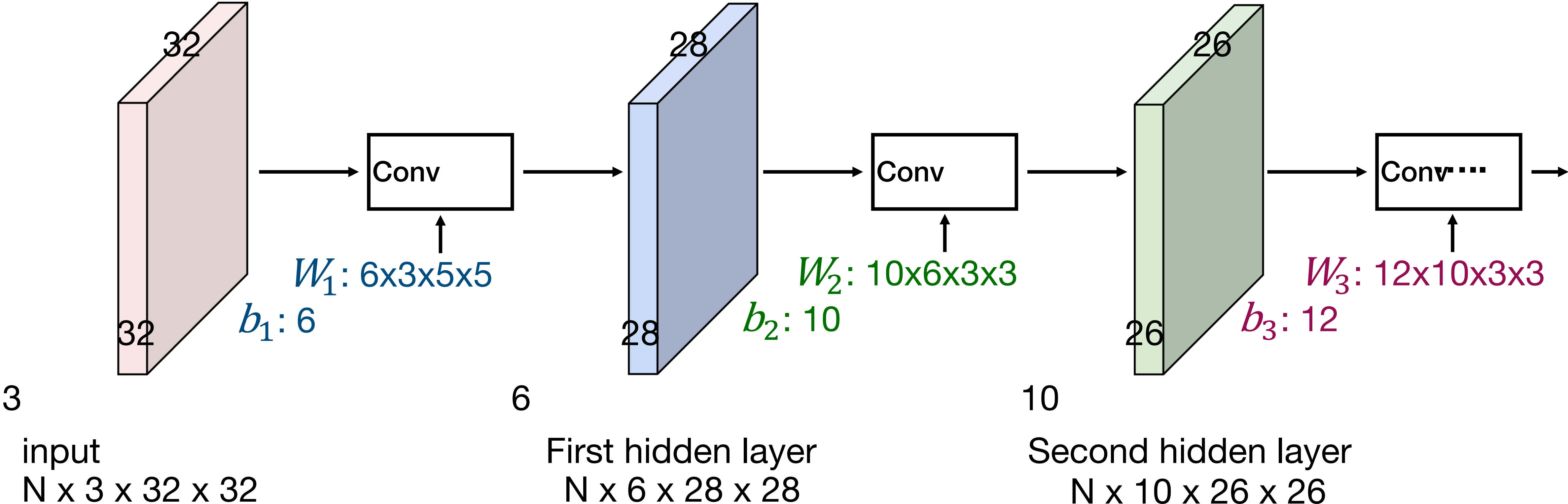




# Stacking Convolutions

Q: What happens if we stack two convolution layers?

(Recall  $y = W_2 W_1 x$  is a linear classifier)



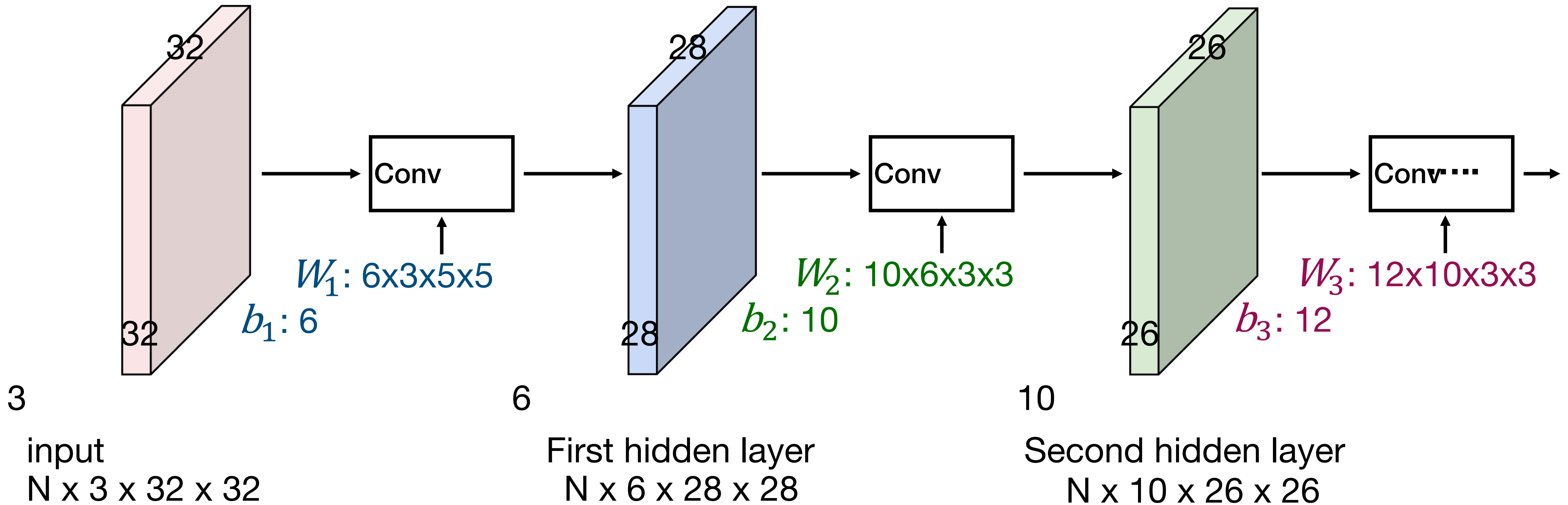


# Stacking Convolutions

**Q:** What happens if we stack two convolution layers?

(Recall  $y = W_2 W_1 x$  is a linear classifier)

**A:** We get another convolution!



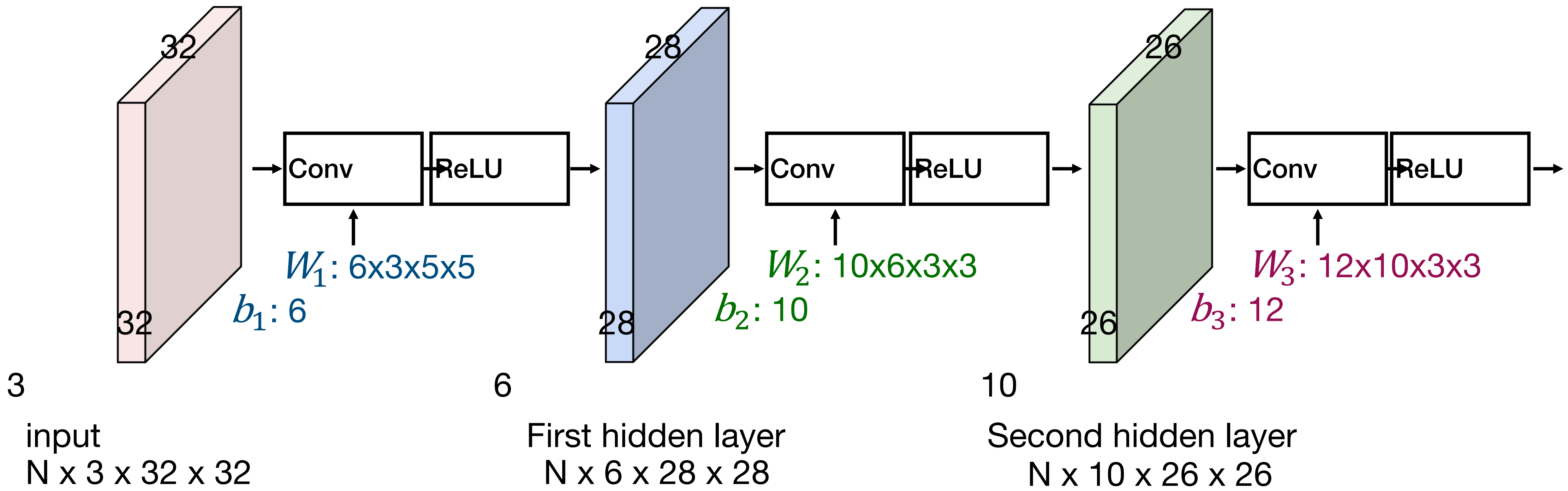


# Stacking Convolutions: insert activation function

**Q:** What happens if we stack two convolution layers?

(Recall  $y = W_2 W_1 x$  is a linear classifier)

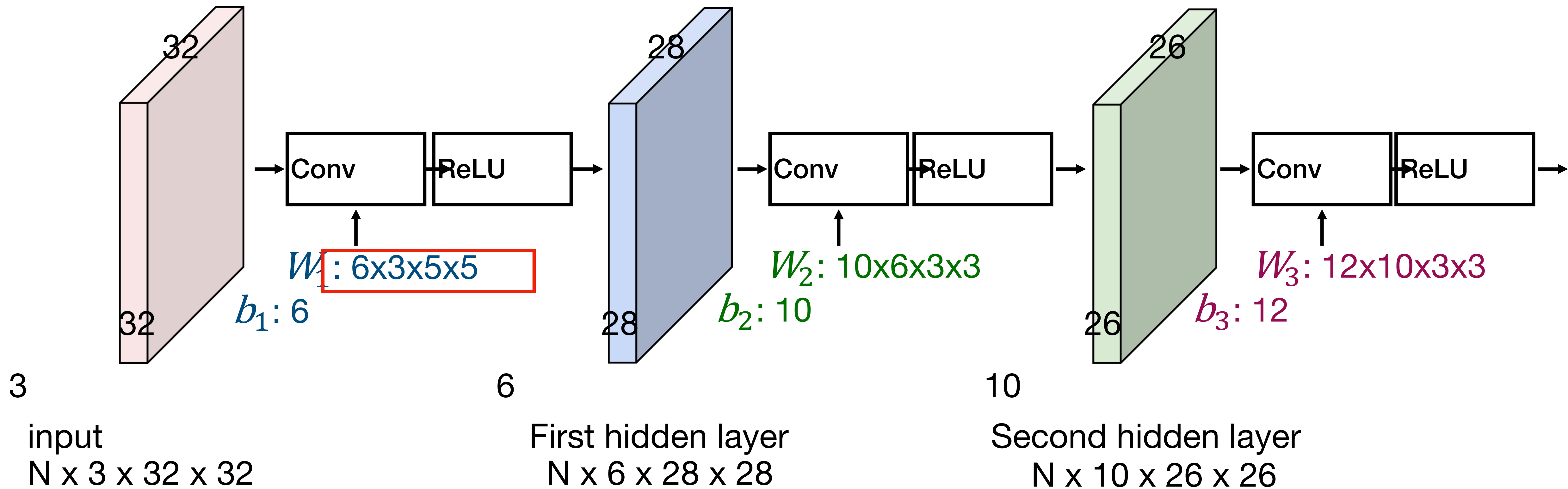
**A:** We get another convolution!



Non-linear relationships

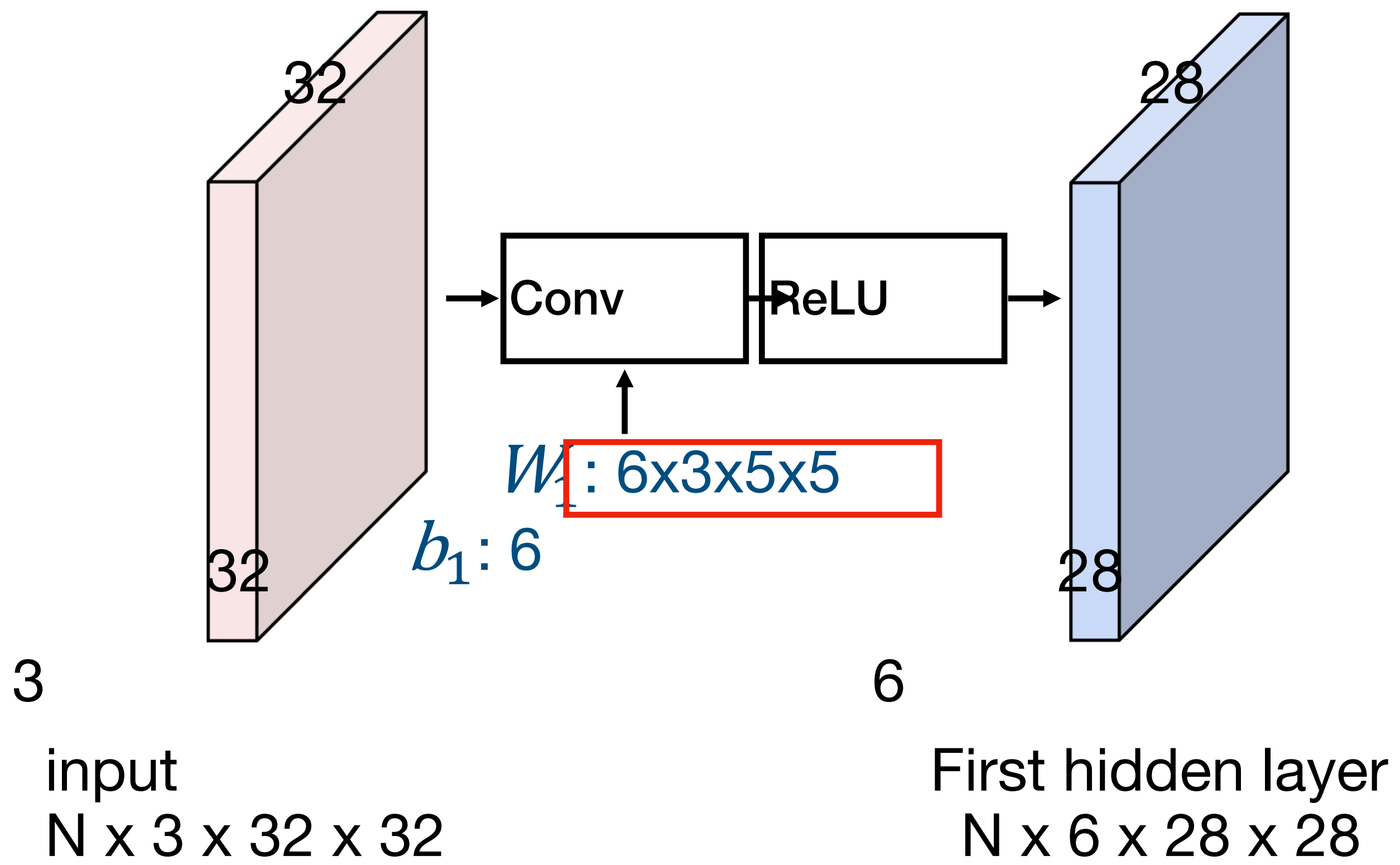


# What do convolutional filters learn?

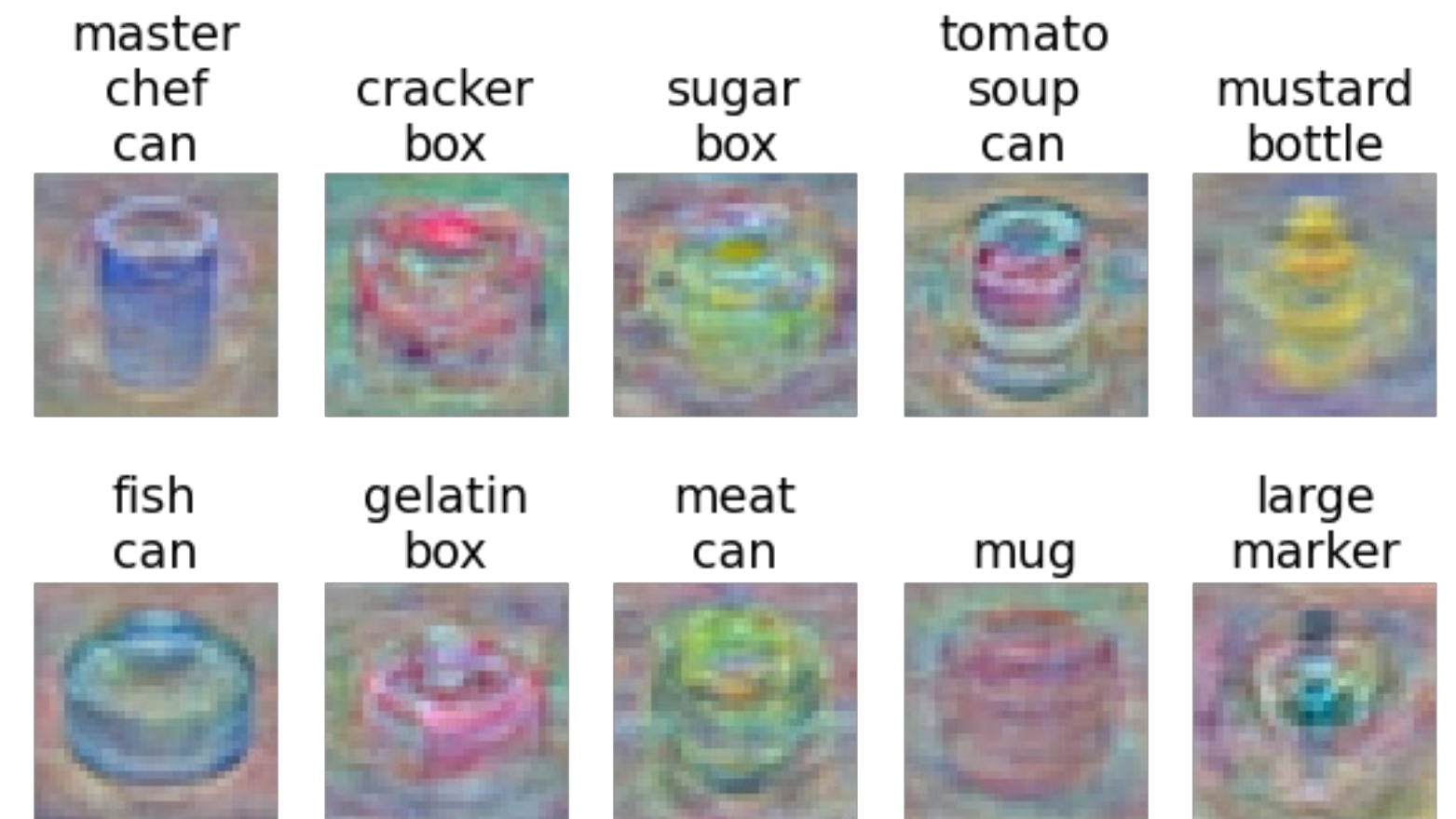




# What do convolutional filters learn?



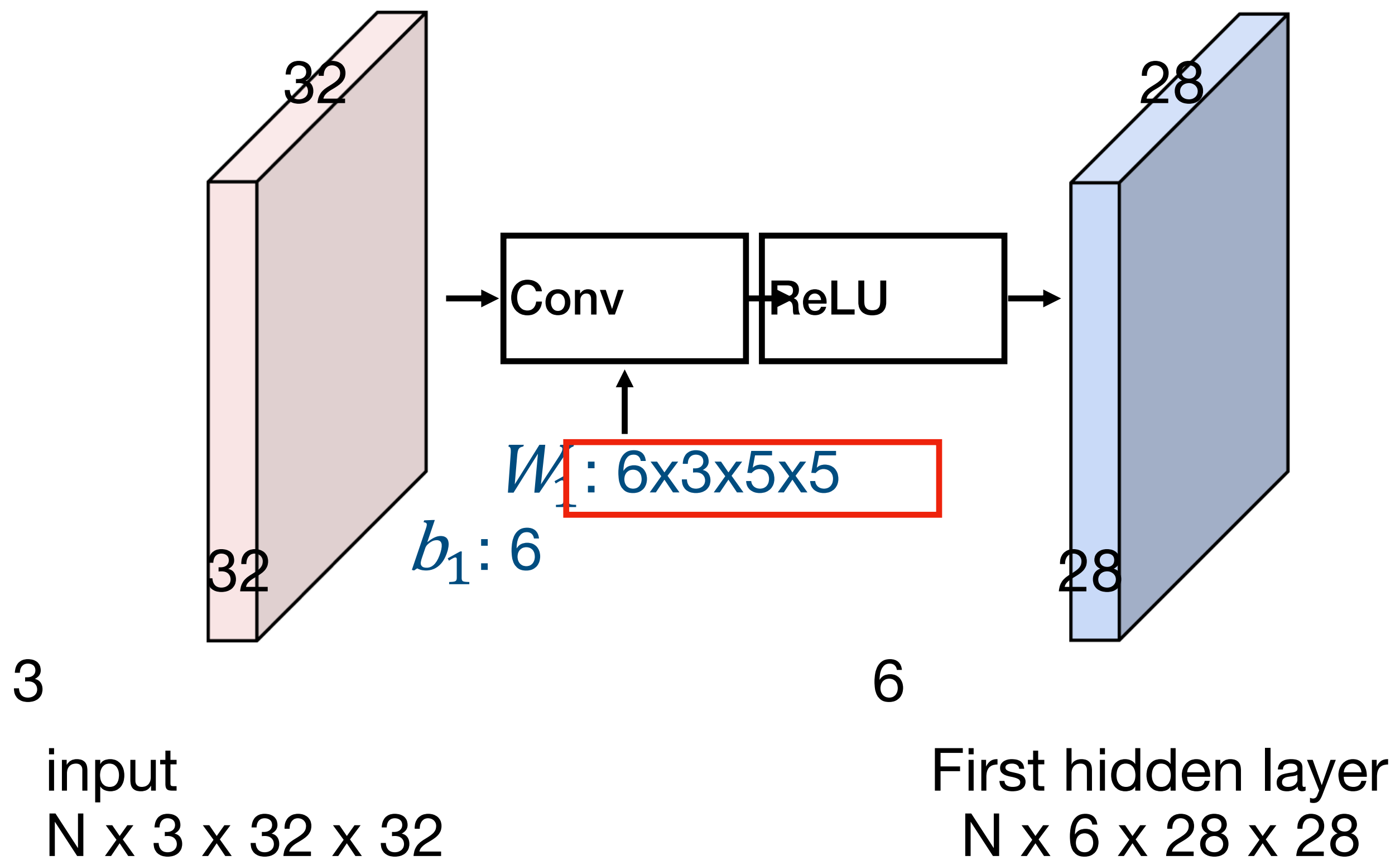
Linear classifier: One template per class







# What do convolutional filters learn?



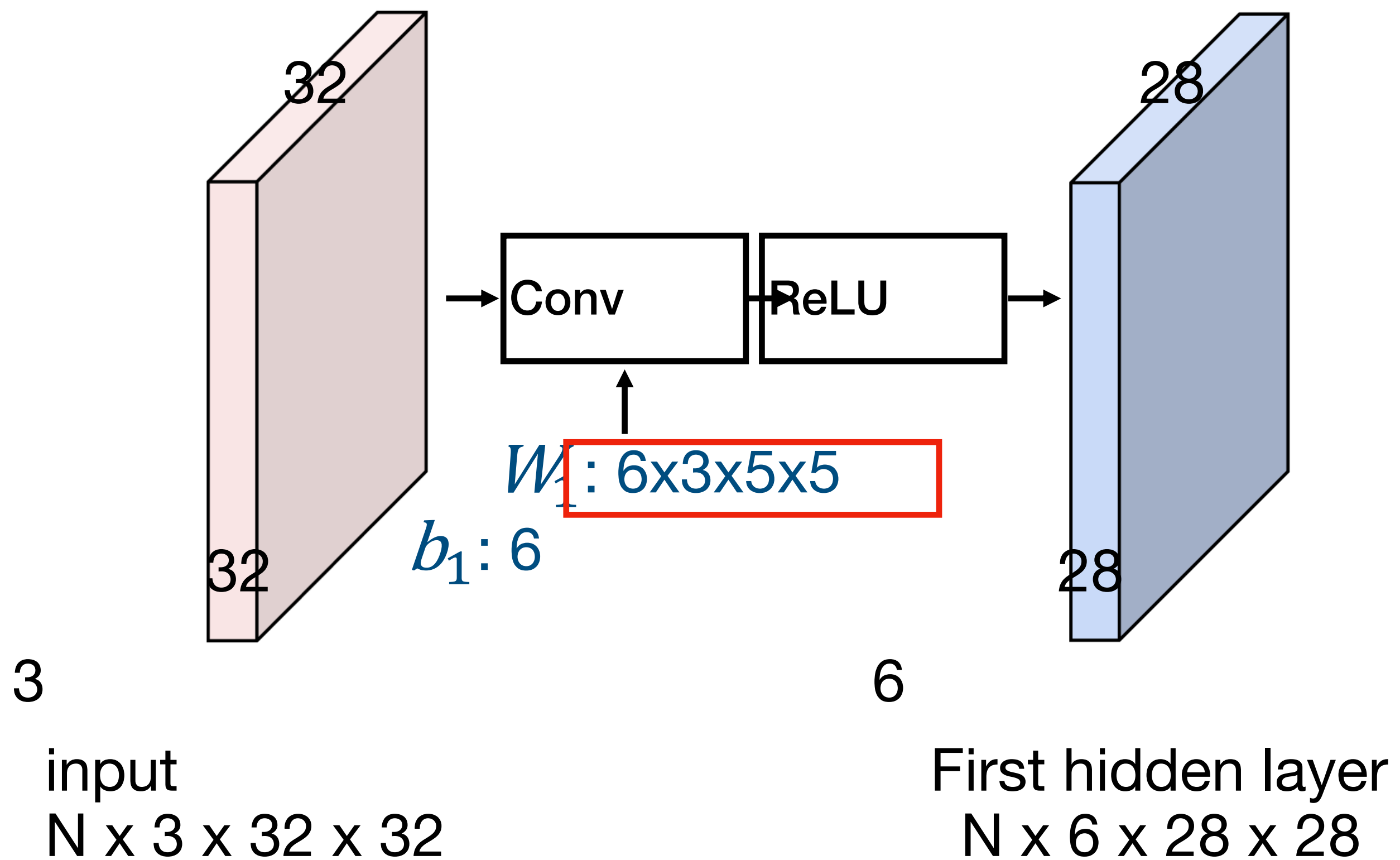
MLP: Bank of whole-image templates



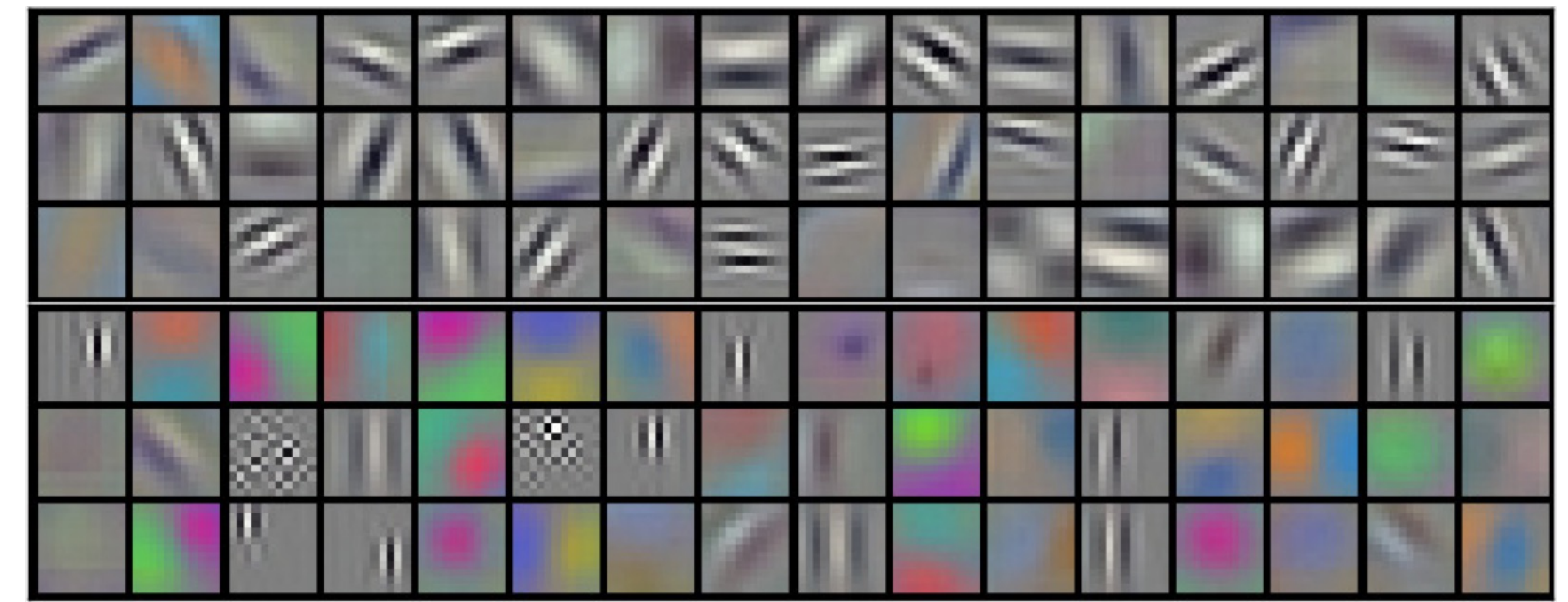
\* Global wrt. the entire image



# What do convolutional filters learn?



First-layer conv filters: local image templates  
(often learns oriented edges, opposing colors)



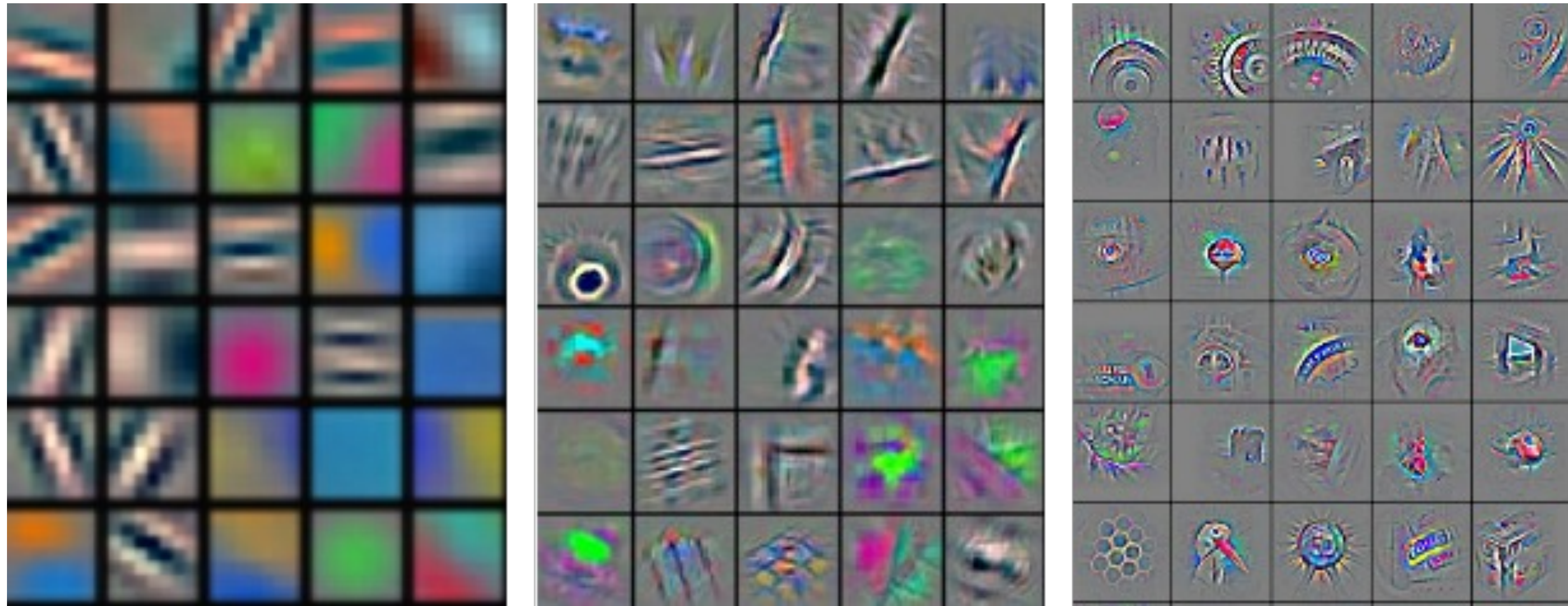
AlexNet: 96 filters, each 3x11x11

\* Local



# What do convolutional filters learn?

## Feature visualization



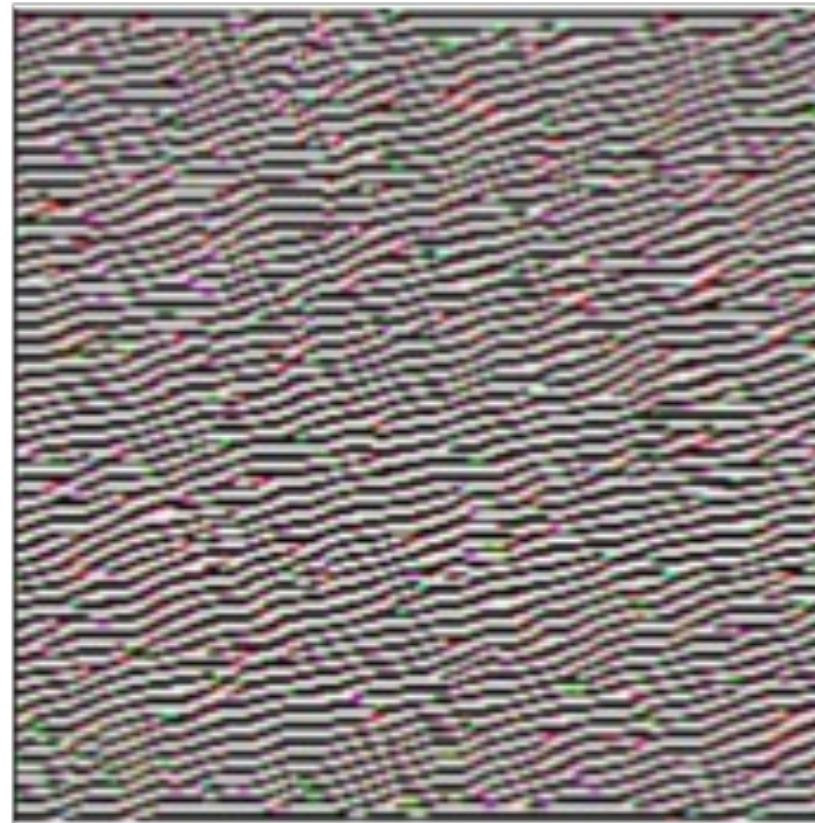


# What do convolutional filters learn?

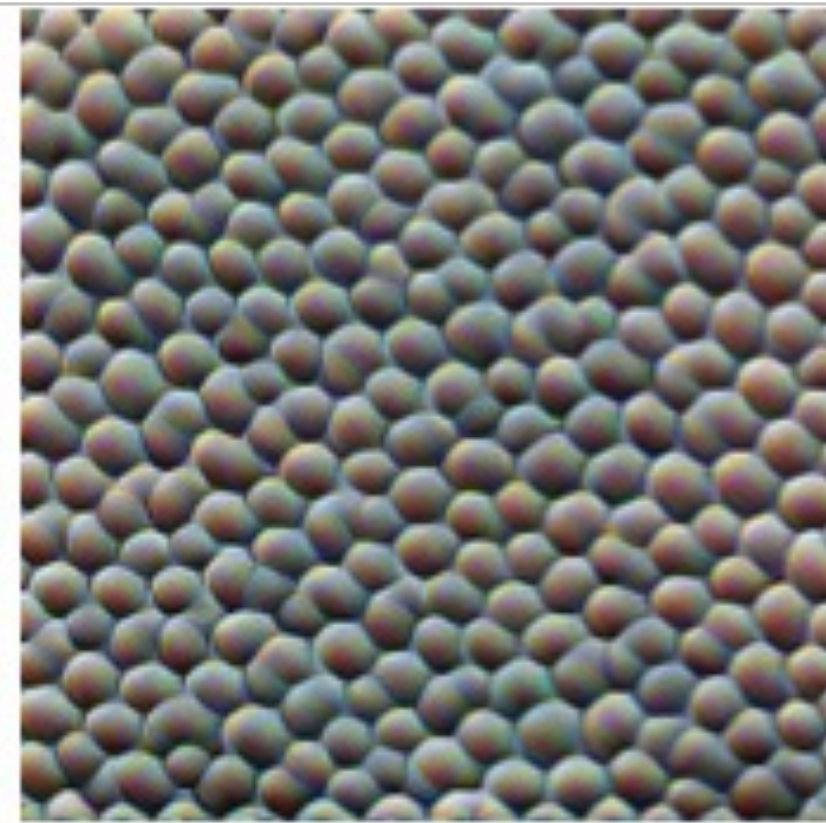
## Feature visualization

distill.pub

Edges



Textures



Patterns



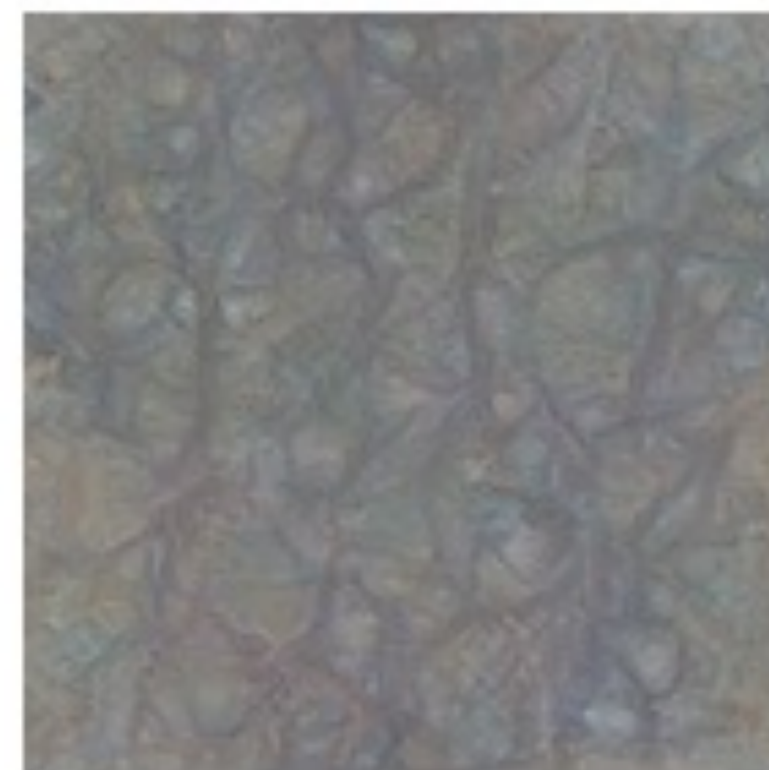
Parts



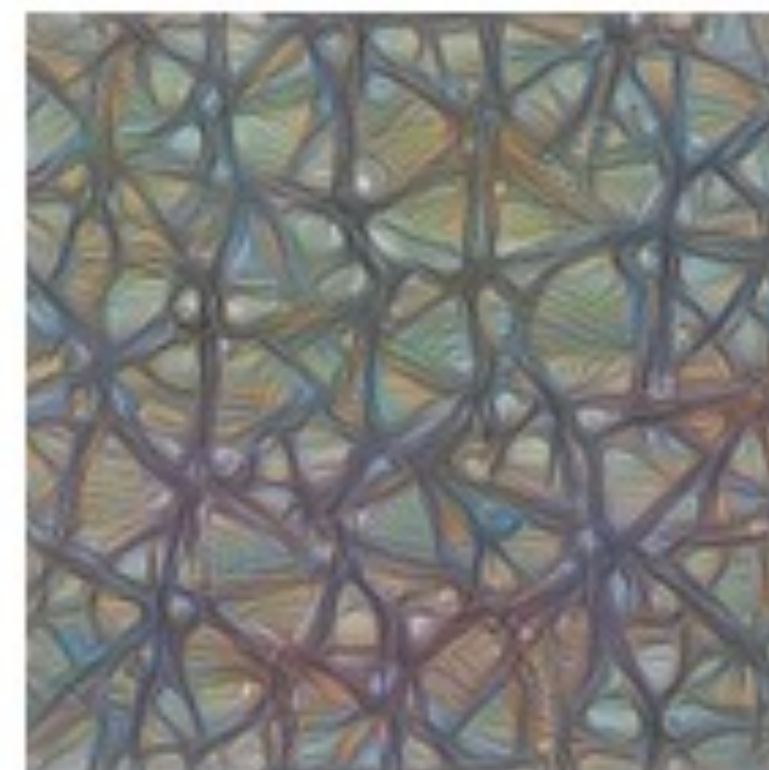
Objects



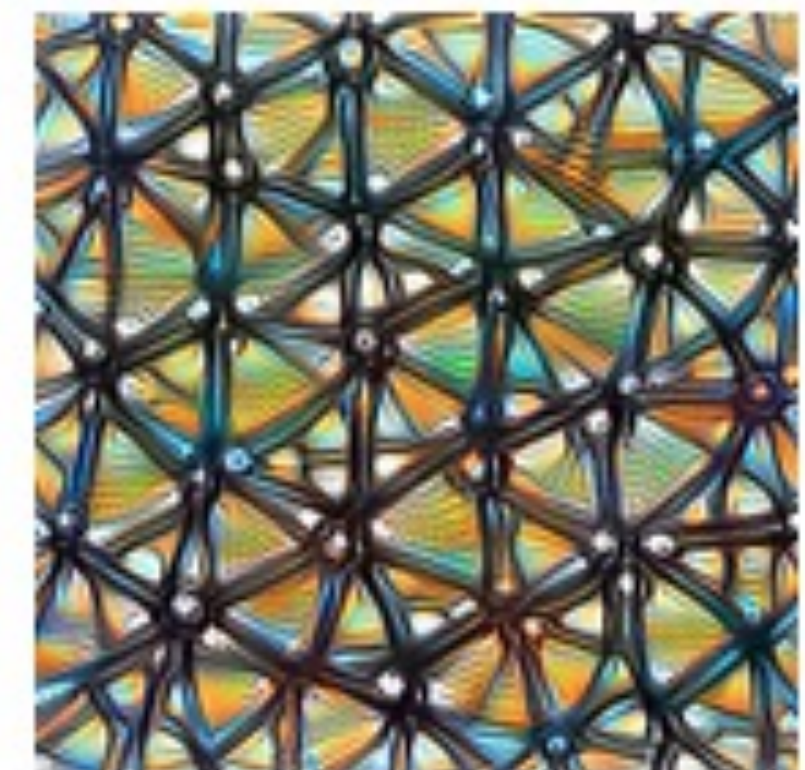
Step 0



Step 4



Step 48



Step 2048




# What do convolutional filters learn?


## Activation mask

<https://christophm.github.io/interpretable-ml-book/cnn-features.html>



 = Human annotated ground truth

 = Top activated area

 = Area of Intersection

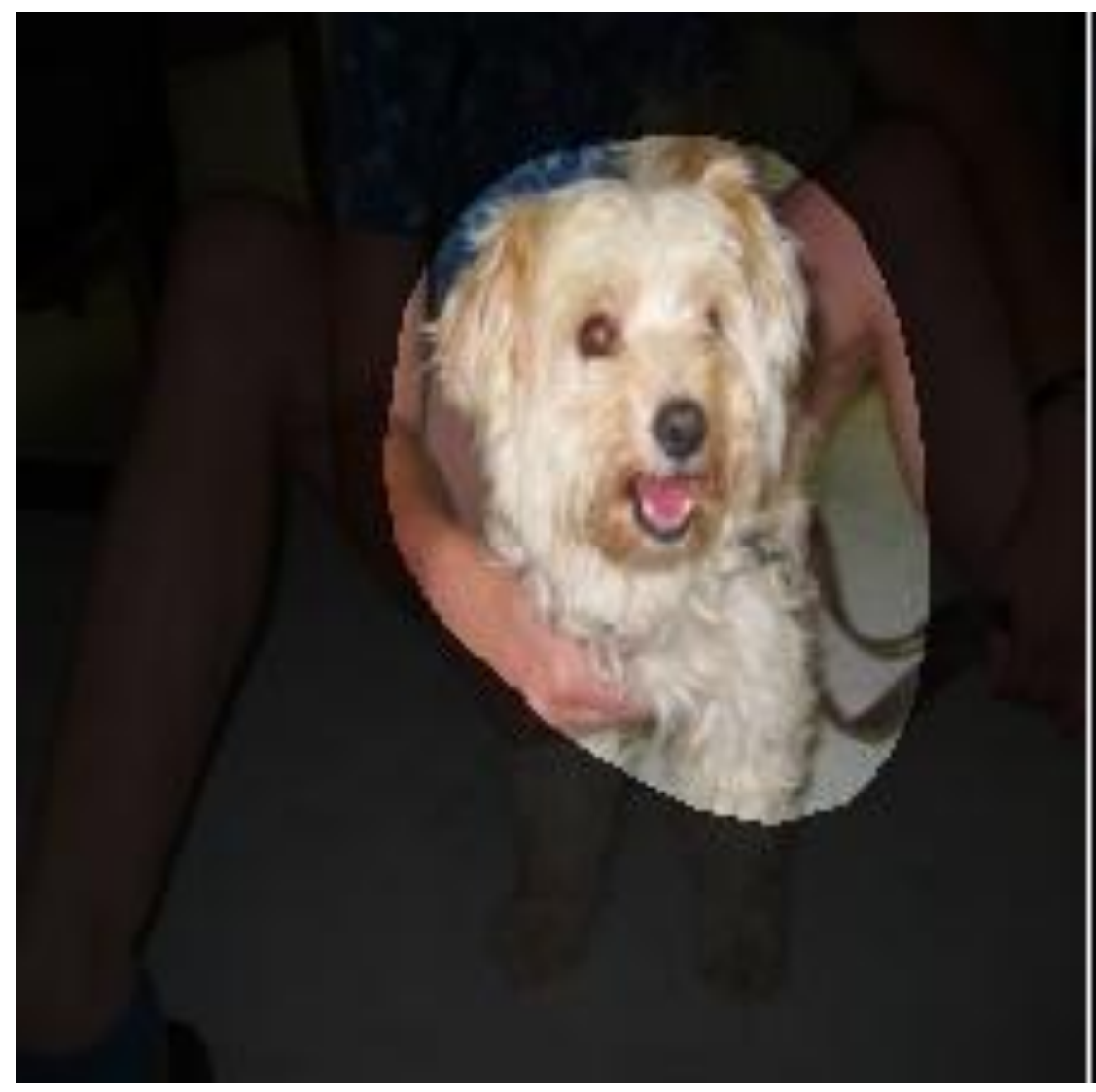
 = Area of Union



What do

learn?

Activation mask



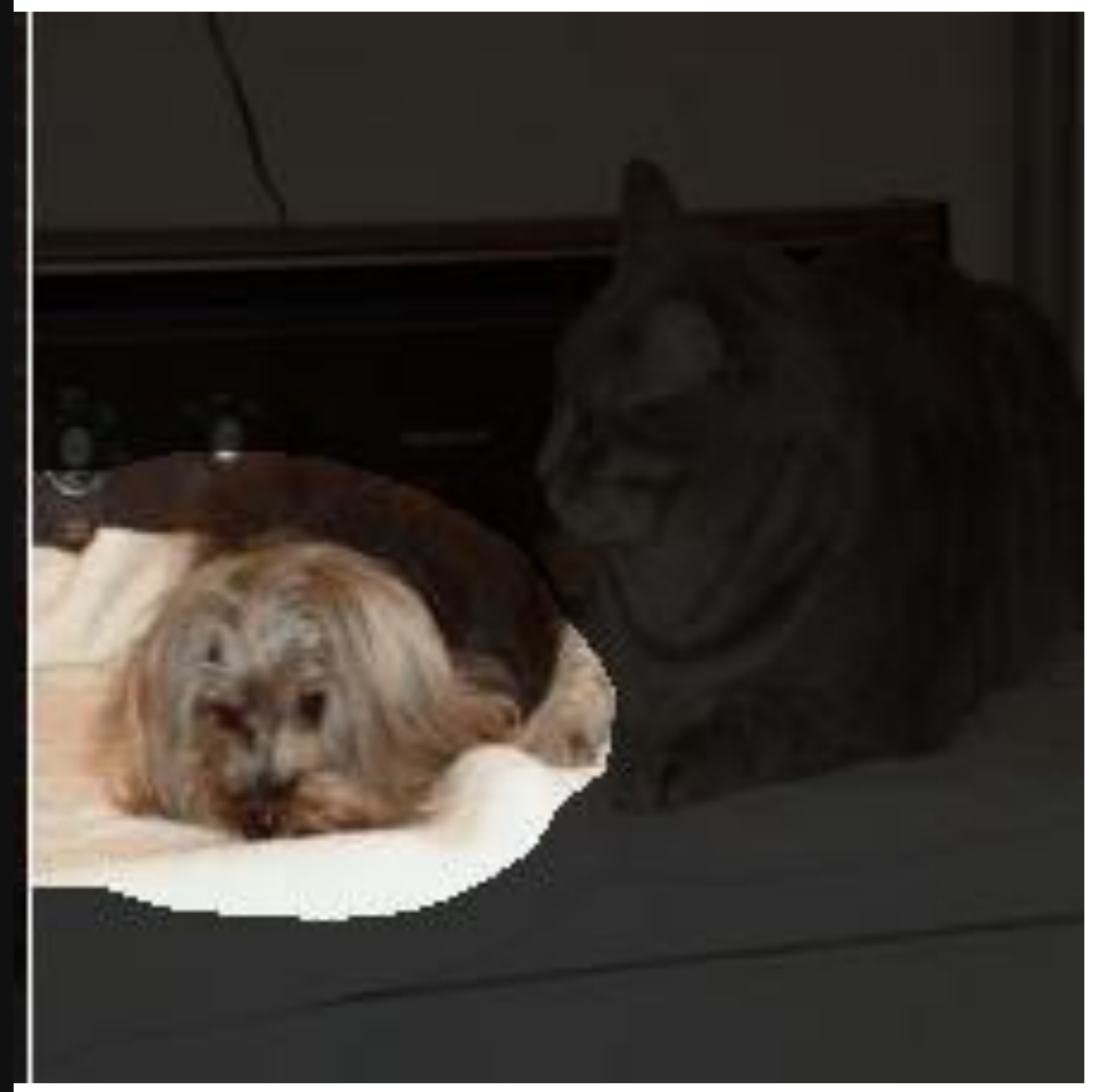
**Interpretable  
Machine Learning**

Second Edition

A Guide for Making  
Black Box Models Explainable

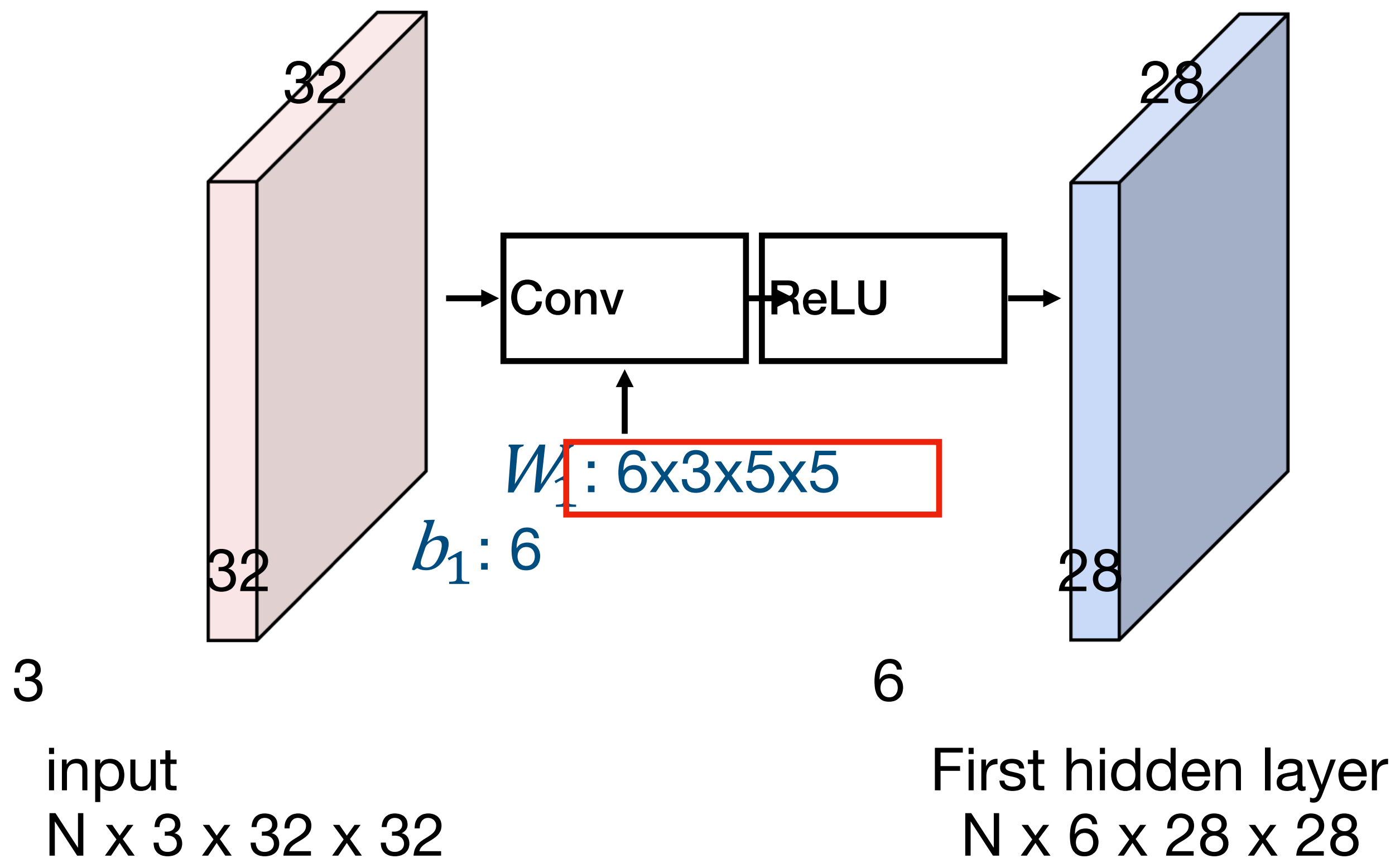
Christoph Molnar

[github.io/interpretable-ml-book/cnn-features.html](https://github.io/interpretable-ml-book/cnn-features.html)



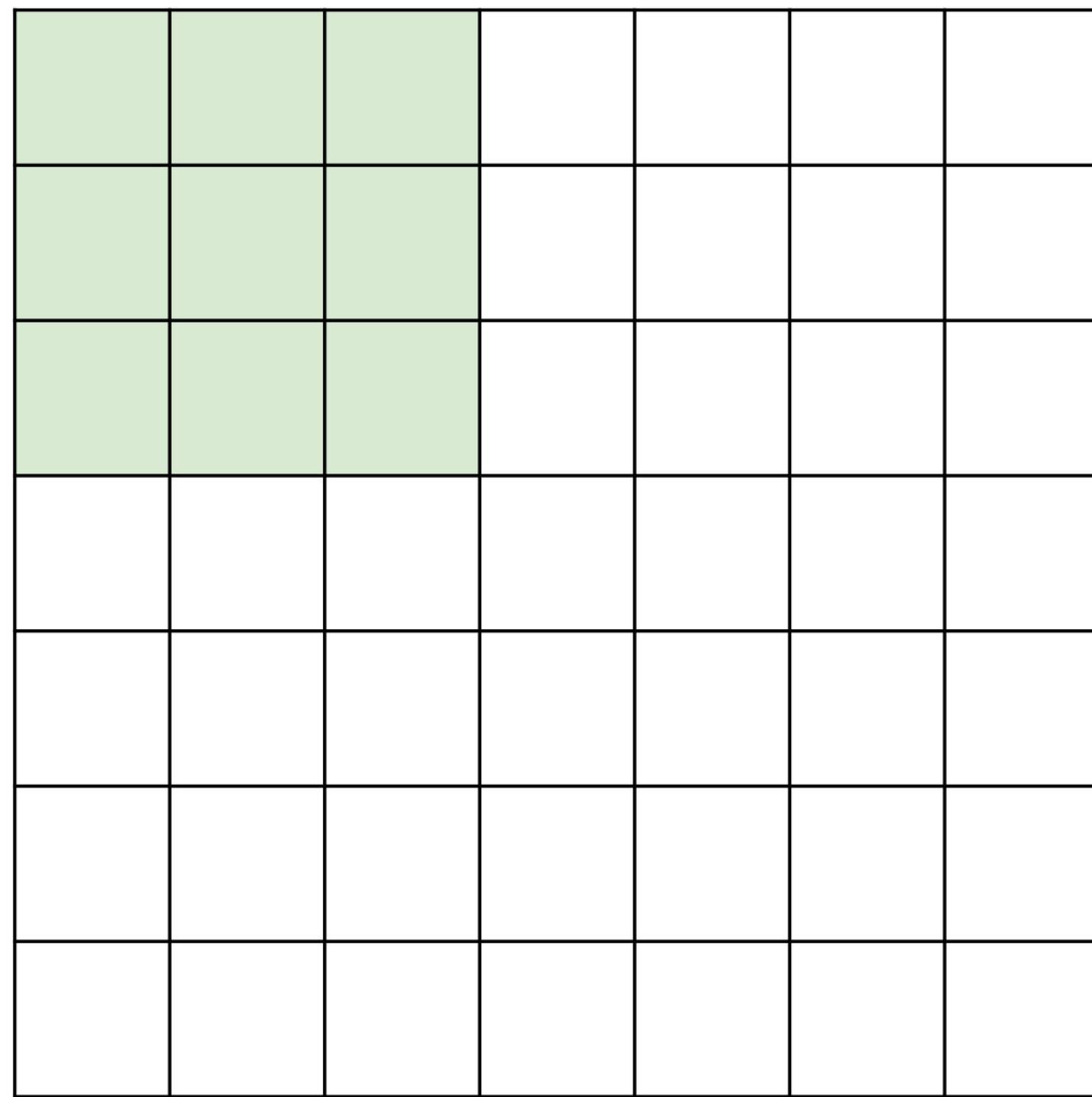


# A closer look at spatial dimensions





# A closer look at spatial dimensions



7

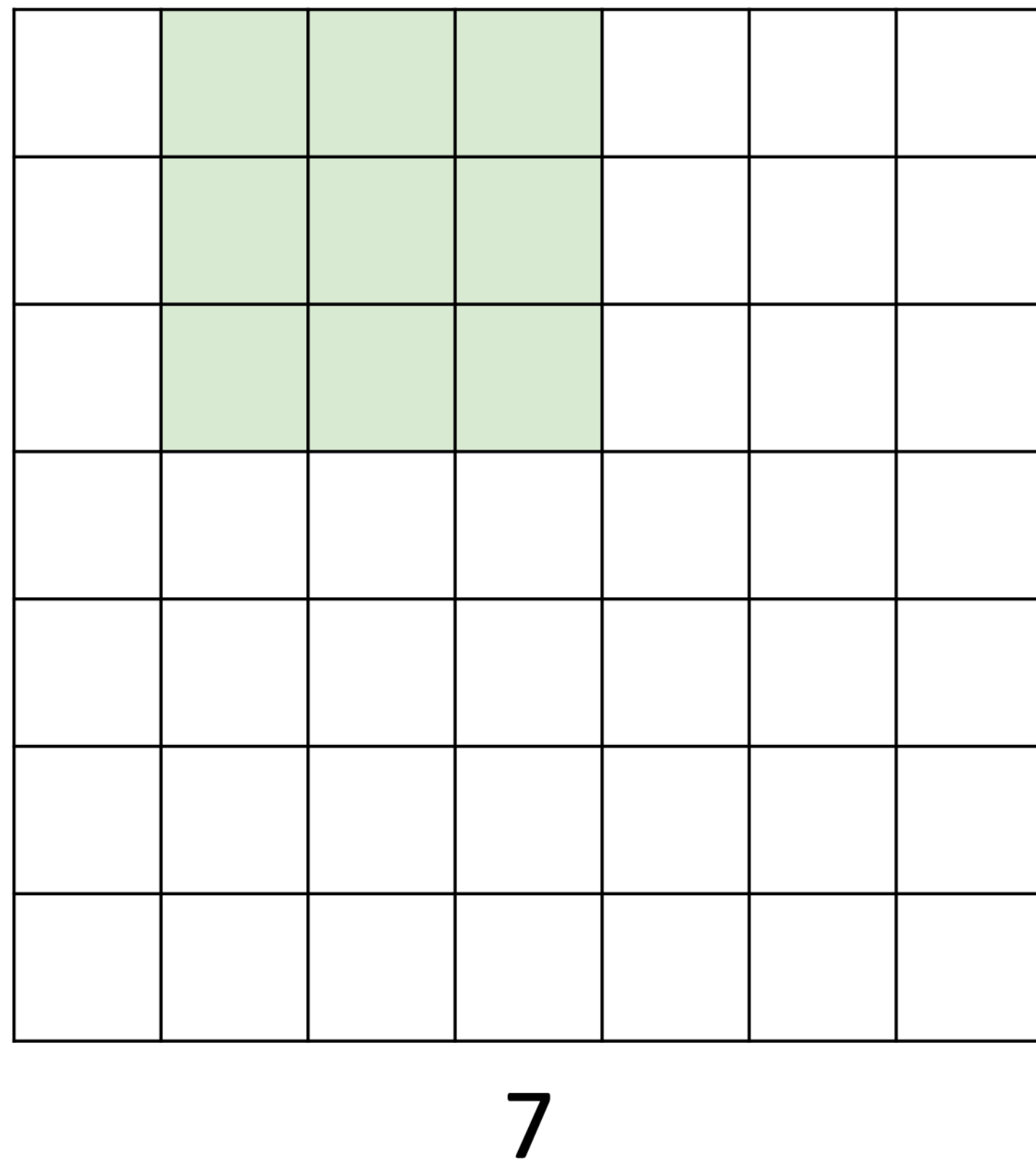
7

Input: 7x7  
Filter: 3x3





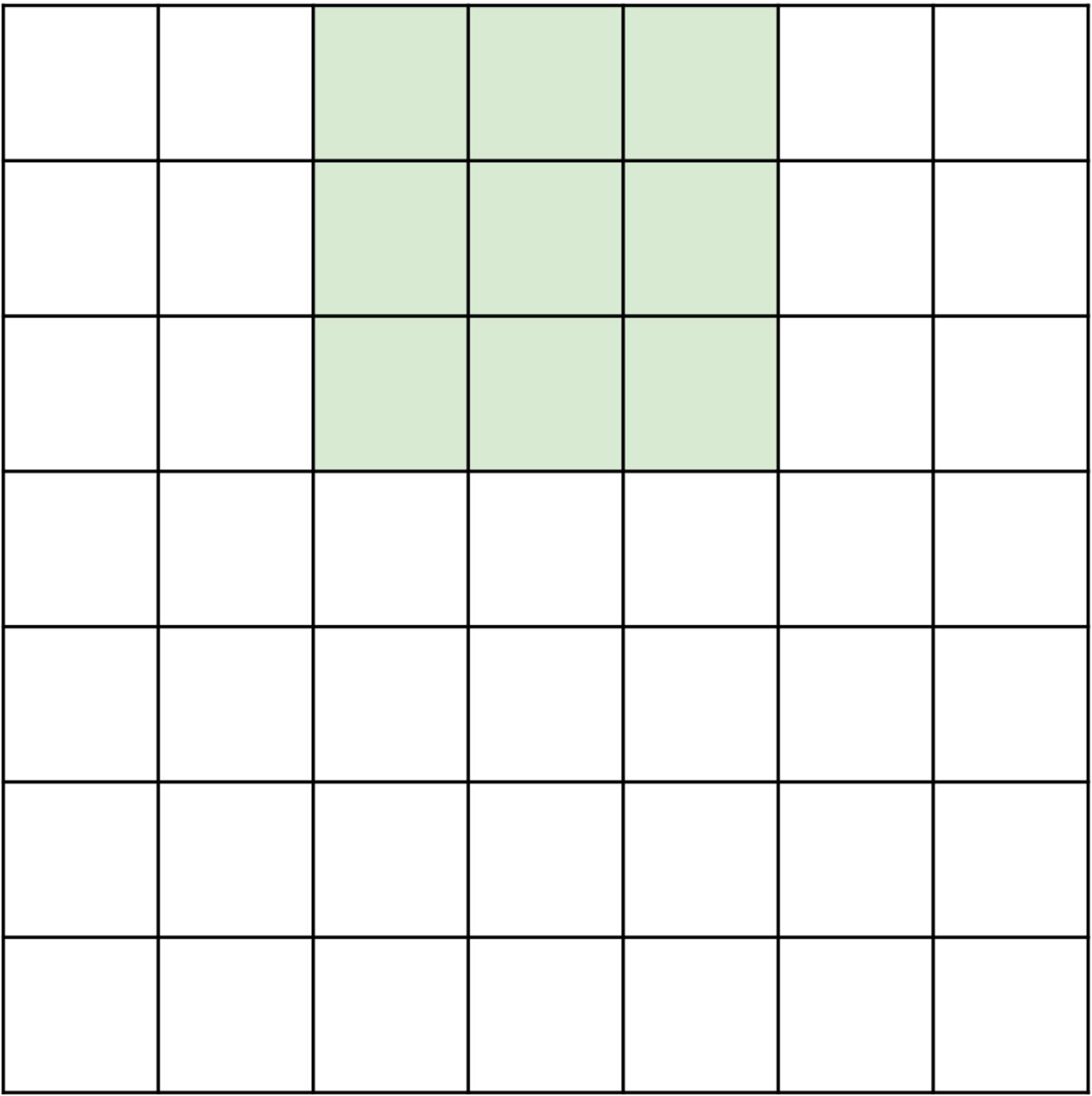
# A closer look at spatial dimensions



Input: 7x7  
Filter: 3x3



# A closer look at spatial dimensions



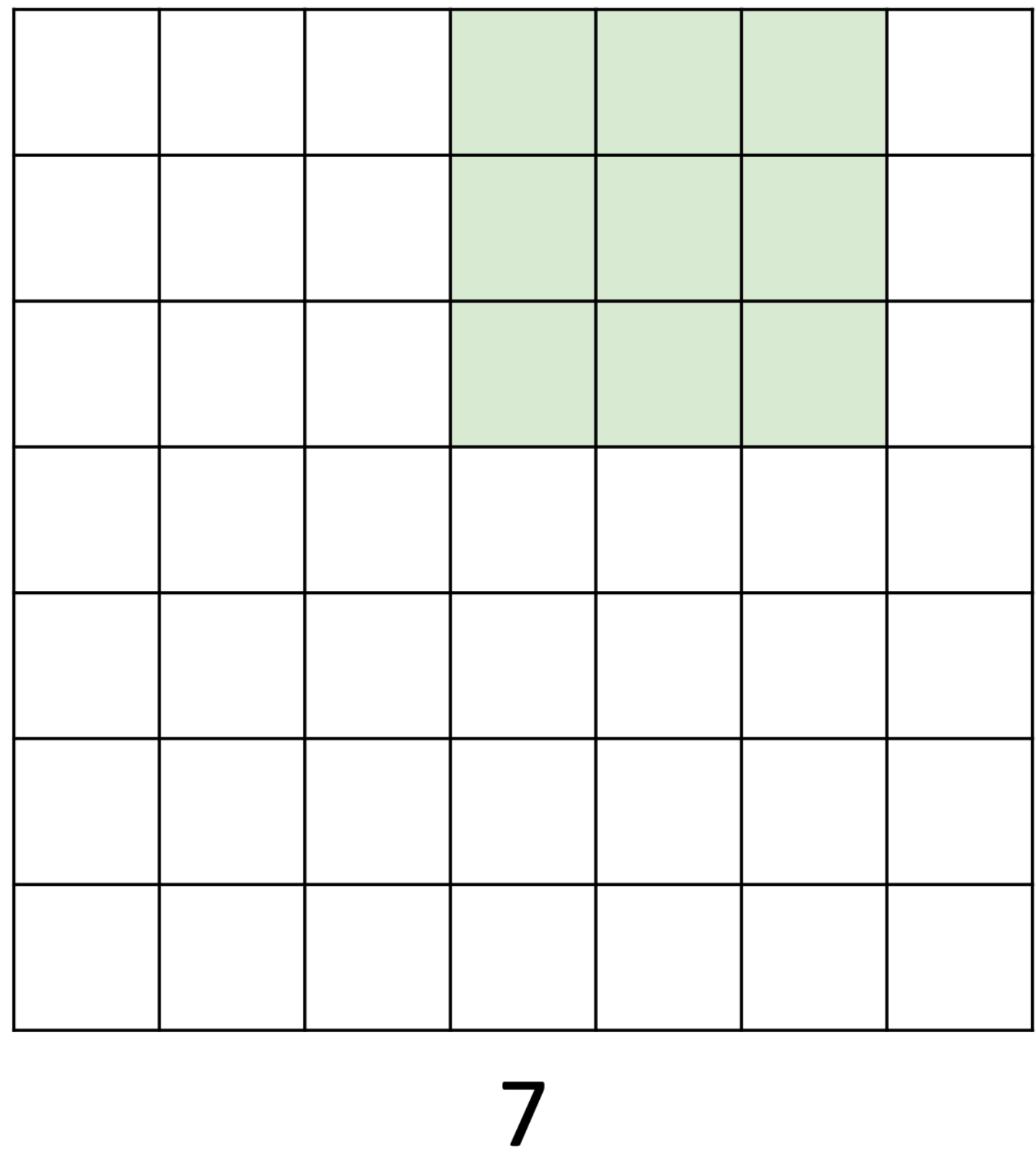
Input: 7x7  
Filter: 3x3

7

7



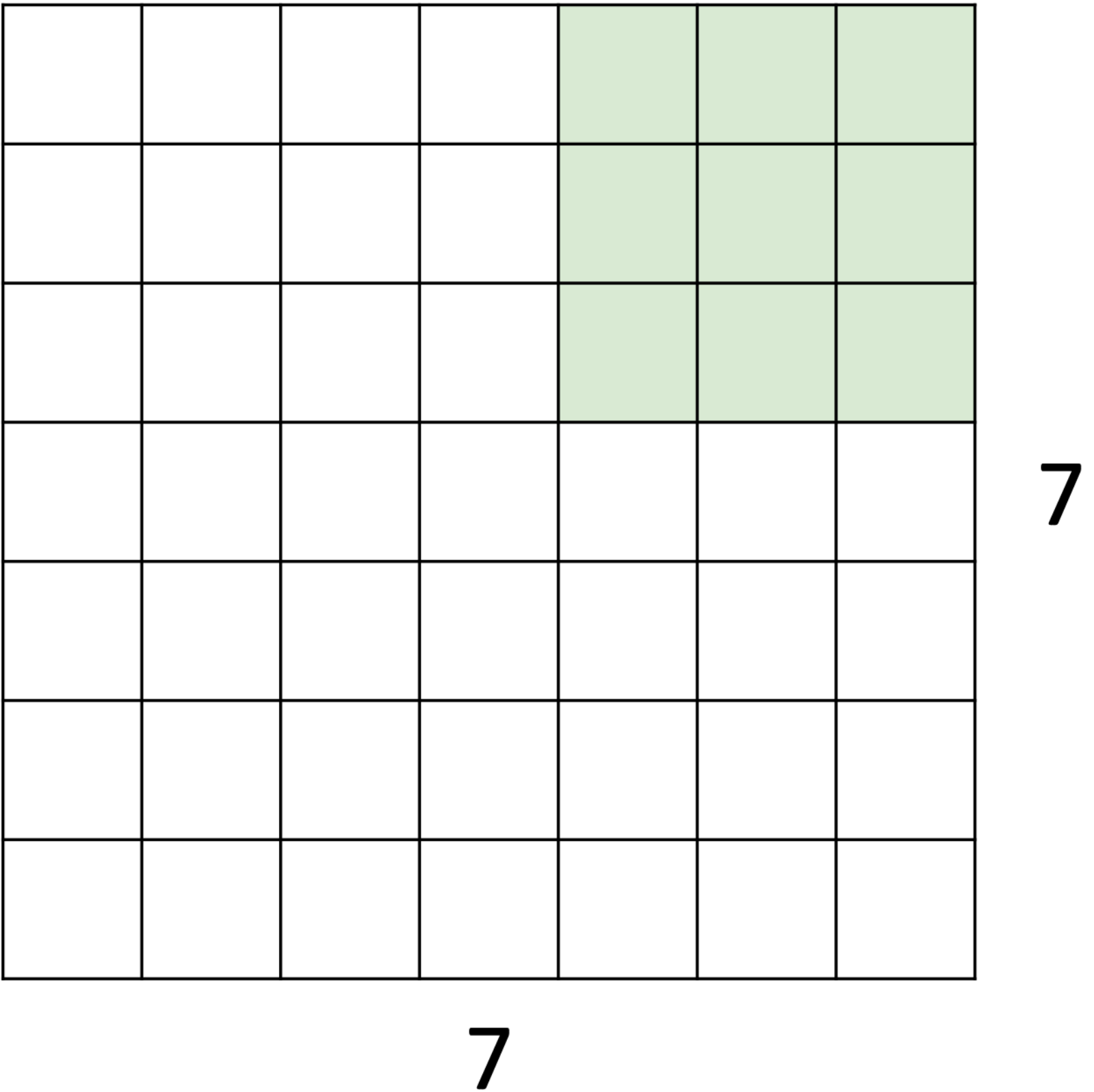
# A closer look at spatial dimensions



Input: 7x7  
Filter: 3x3



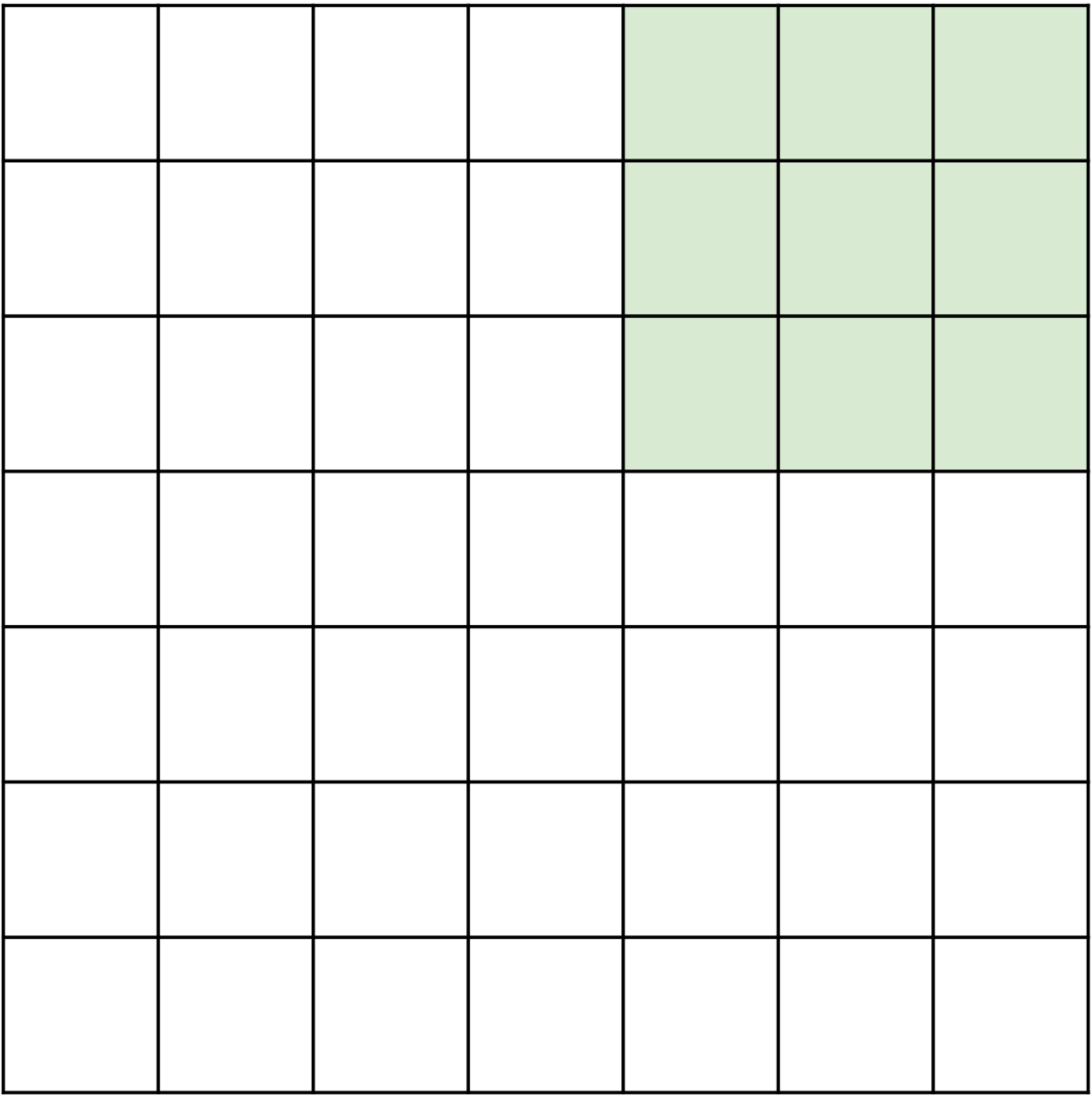
# A closer look at spatial dimensions



Input: 7x7  
Filter: 3x3  
Output: 5x5



# A closer look at spatial dimensions



Input: 7x7

Filter: 3x3

Output: 5x5

In general:      **Problem: Feature maps “shrink” with each layer!**  
Input:  $W$   
Filter:  $K$   
Output:  $W - K + 1$



# A closer look at spatial dimensions

0	0	0	0	0	0	0	0	0
0								0
0								0
0								0
0								0
0								0
0								0
0								0
0								0
0	0	0	0	0	0	0	0	0

Input: 7x7

Filter: 3x3

Output: 5x5

In general:

Input:  $W$

Filter:  $K$

Output:  $W - K + 1$

Problem: Feature maps “shrink” with each layer!

Solution: padding

Add zeros around the input



# A closer look at spatial dimensions

0	0	0	0	0	0	0	0	0
0								0
0								0
0								0
0								0
0								0
0								0
0								0
0								0
0	0	0	0	0	0	0	0	0

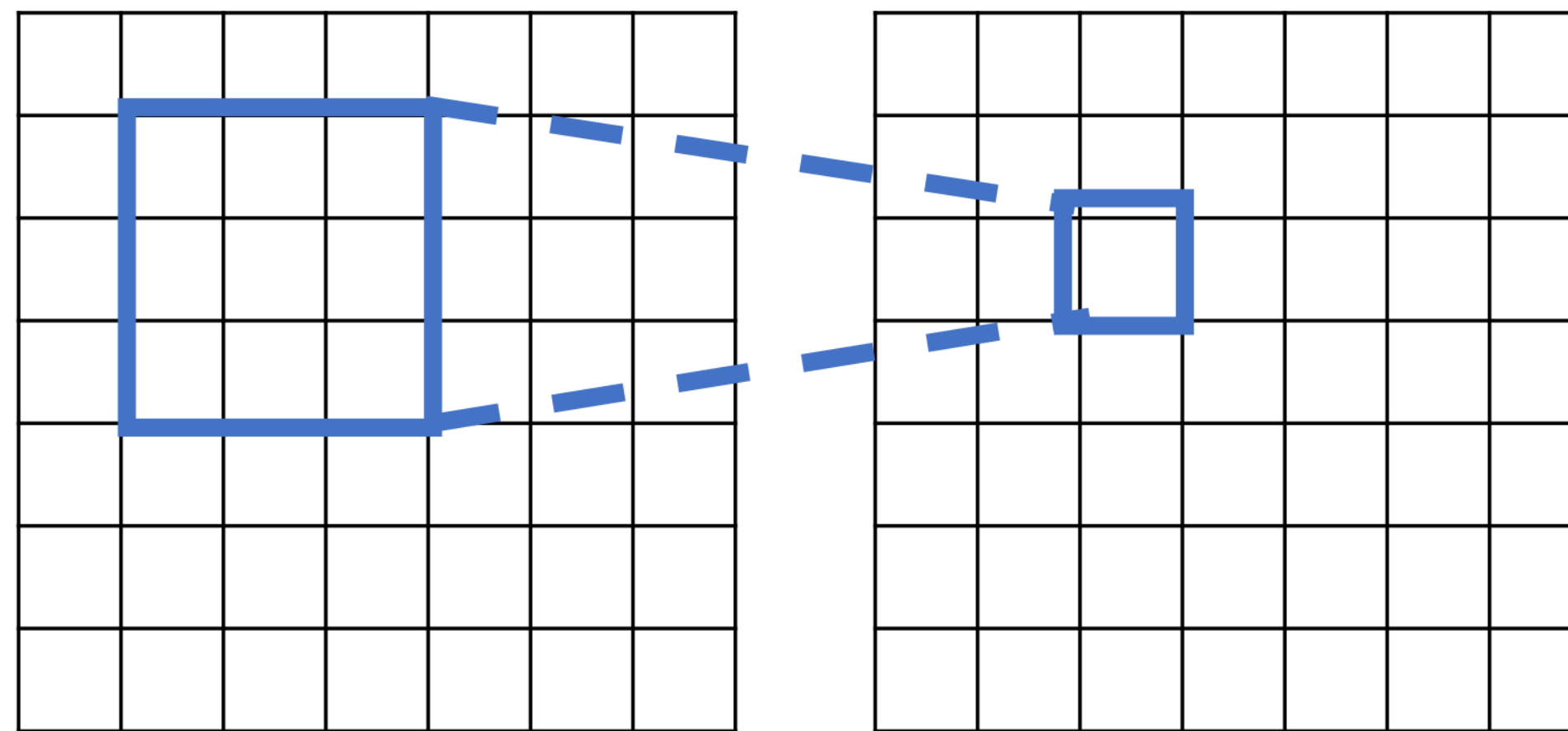
Input: 7x7  
Filter: 3x3  
Output: 5x5

In general:      Very common:  
Input: W            Set  $P = (K - 1) / 2$  to  
Filter: K            make output have  
Padding: P           same size as input!  
Output:  $W - K + 1 + 2P$



# Receptive Fields

For convolution with kernel size  $K$ , each element in the output depends on a  $K \times K$  **receptive field** in the input



Input

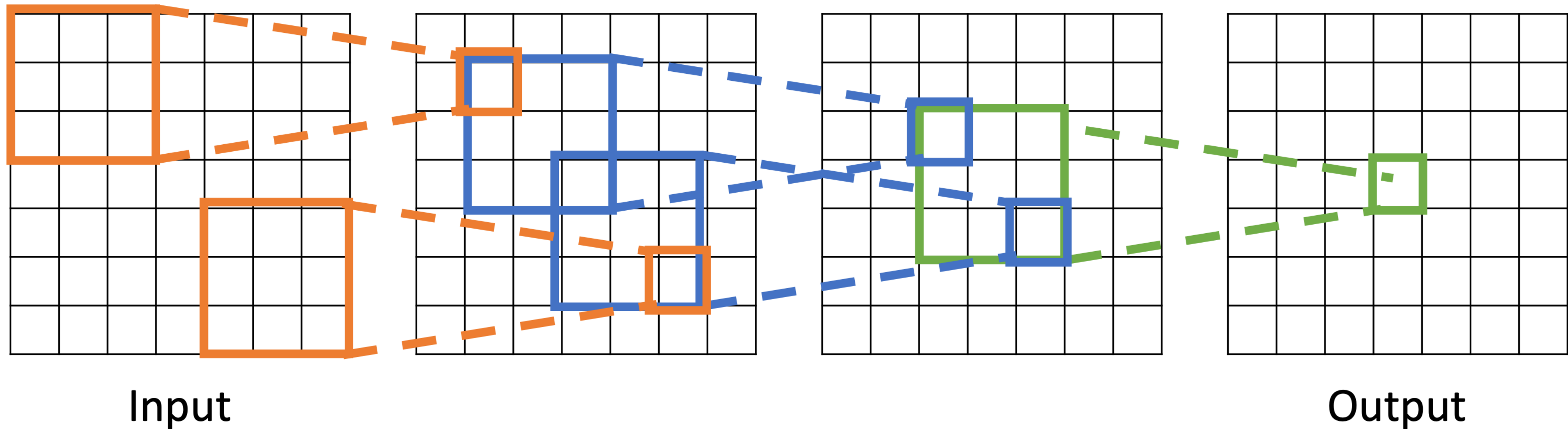
Output





# Receptive Fields

Each successive convolution adds  $K - 1$  to the receptive field size  
With  $L$  layers the receptive field size is  $1 + L * (K - 1)$

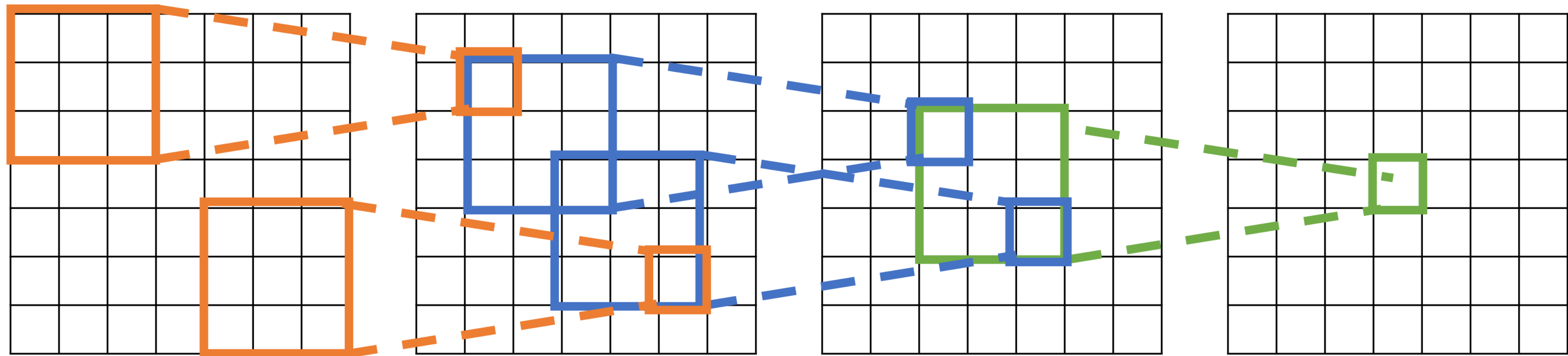


Be careful – “receptive field in the input” vs “receptive field in the previous layer”  
Hopefully clear from context!



# Receptive Fields

Each successive convolution adds  $K - 1$  to the receptive field size  
With  $L$  layers the receptive field size is  $1 + L * (K - 1)$



Input

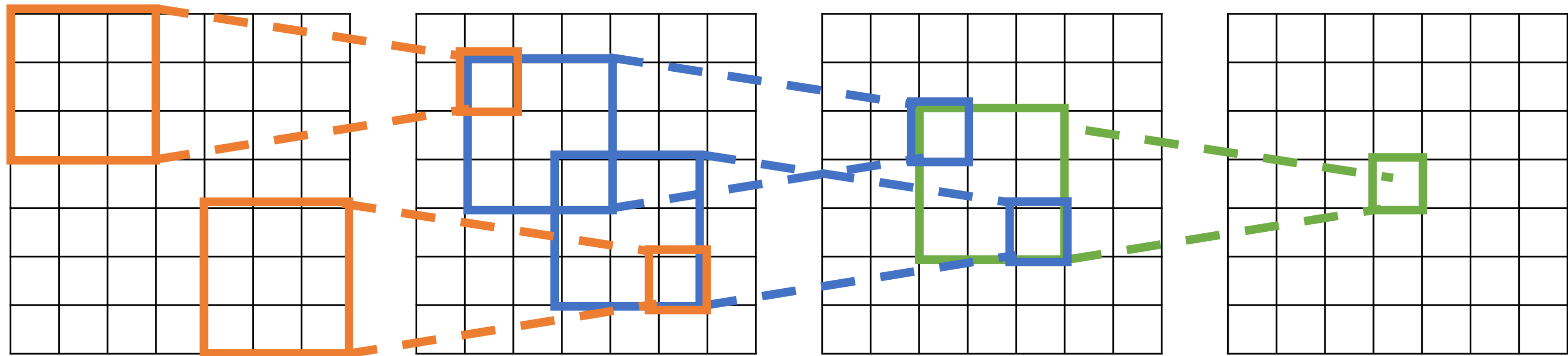
Problem: For large images we need many layers for each output to “see” the whole image

Output



# Receptive Fields

Each successive convolution adds  $K - 1$  to the receptive field size  
With  $L$  layers the receptive field size is  $1 + L * (K - 1)$



Input

Output

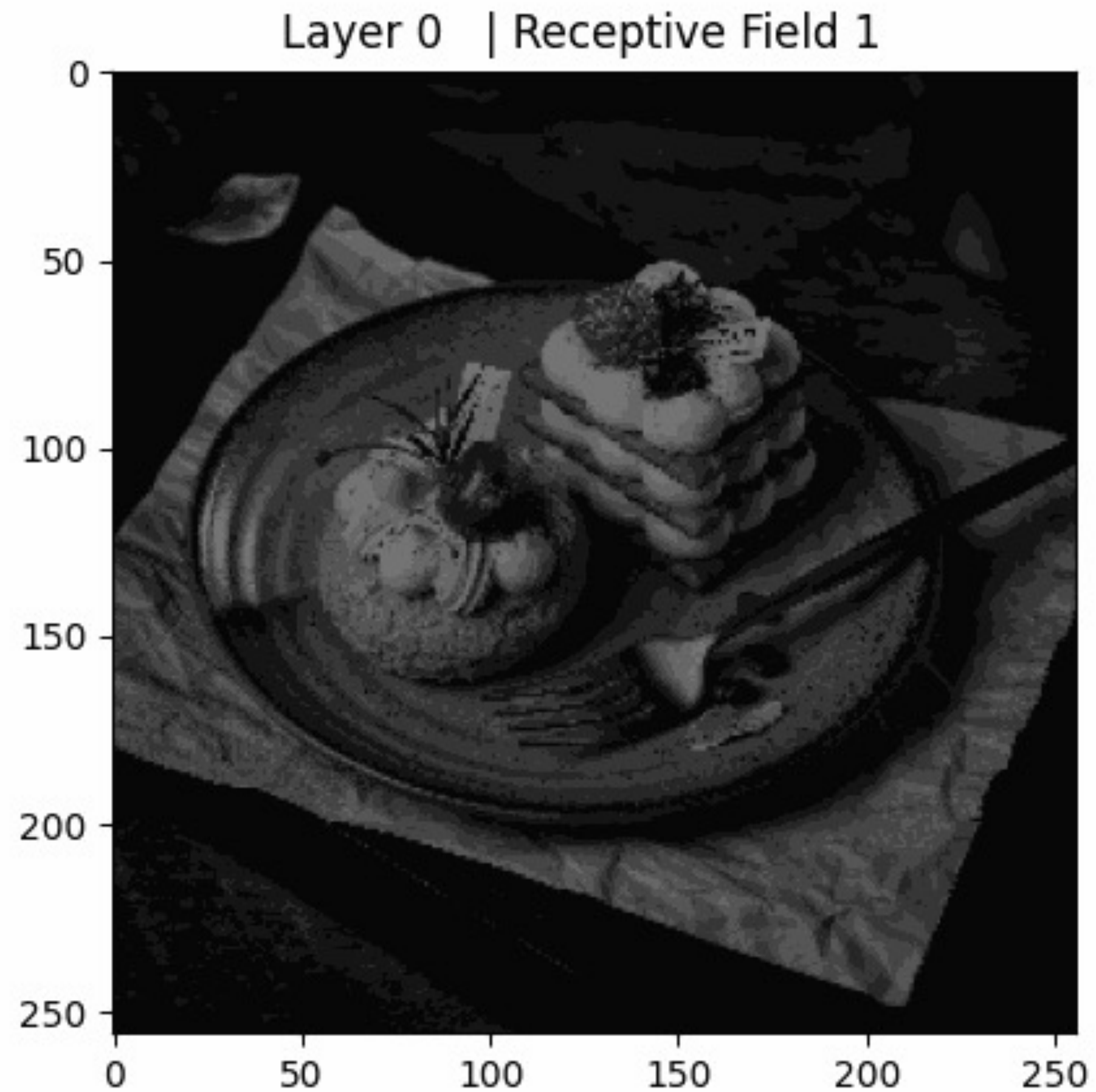
**Problem:** For large images we need many layers for each output to “see” the whole image

**Solution:** Downsample inside the network



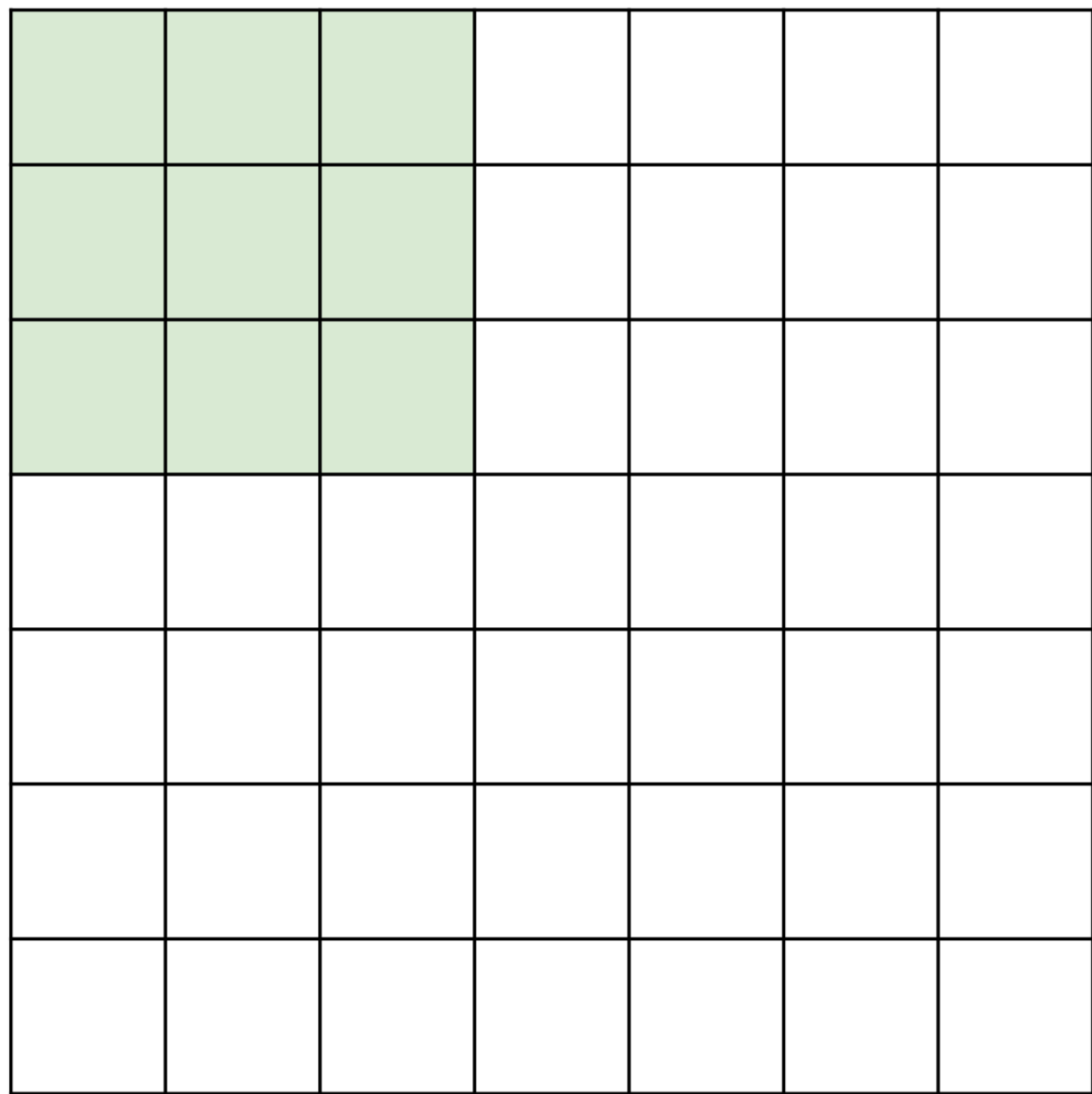
# Receptive Fields

<https://github.com/Fangyh09/pytorch-receptive-field>





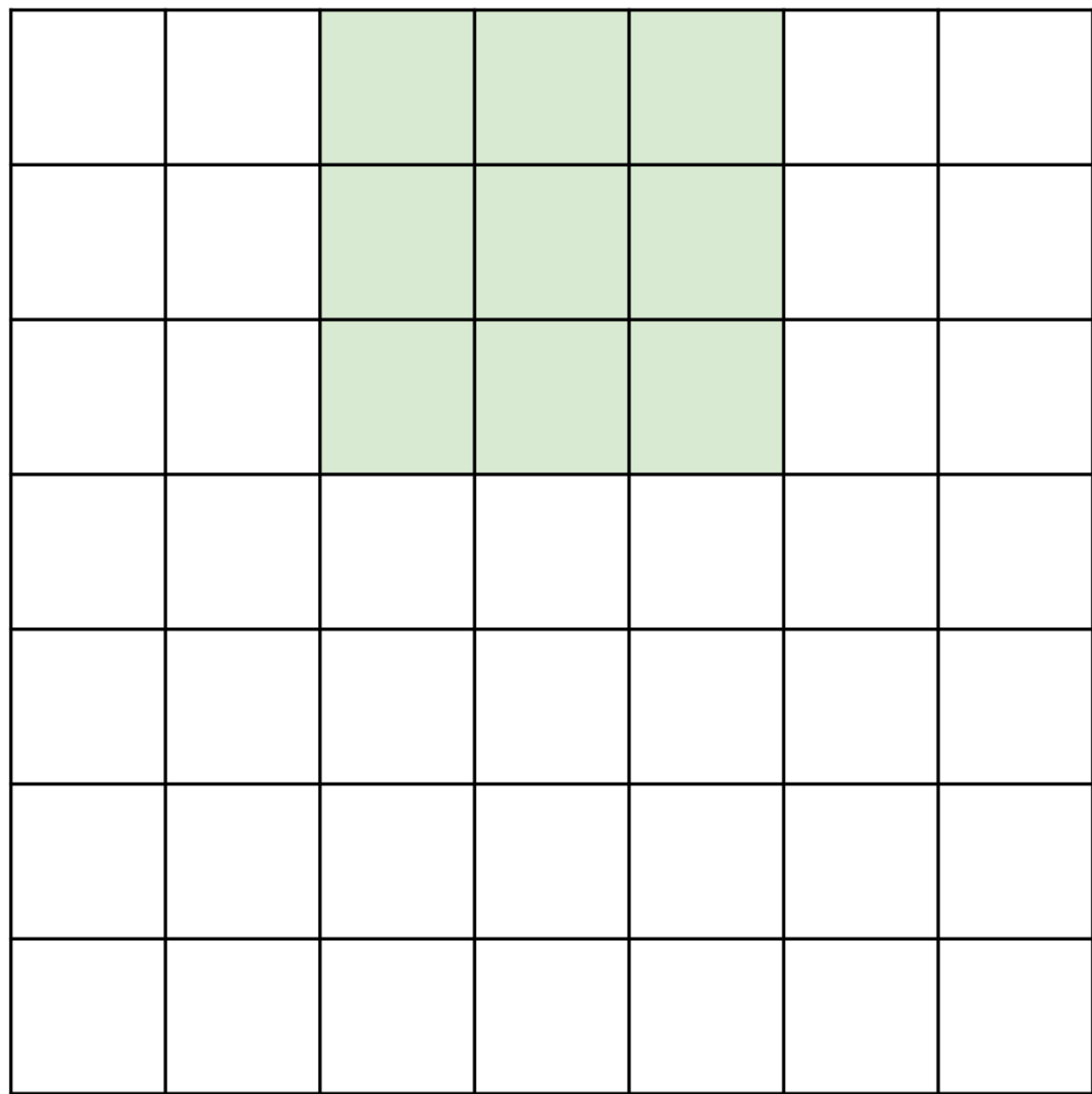
# Strided Convolution



Input: 7x7  
Filter: 3x3  
Stride: 2



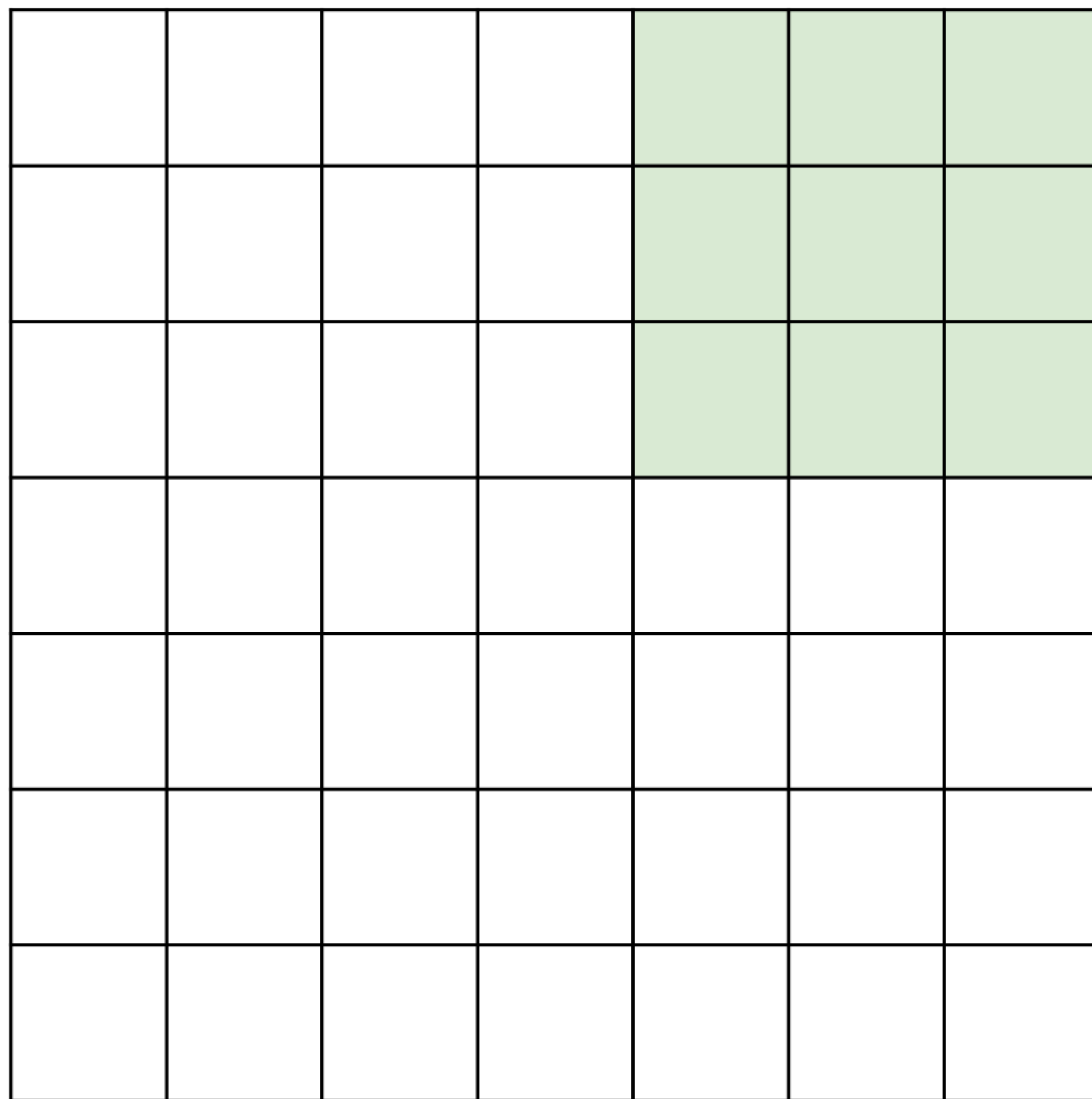
# Strided Convolution



Input: 7x7  
Filter: 3x3  
Stride: 2



# Strided Convolution



Input: 7x7

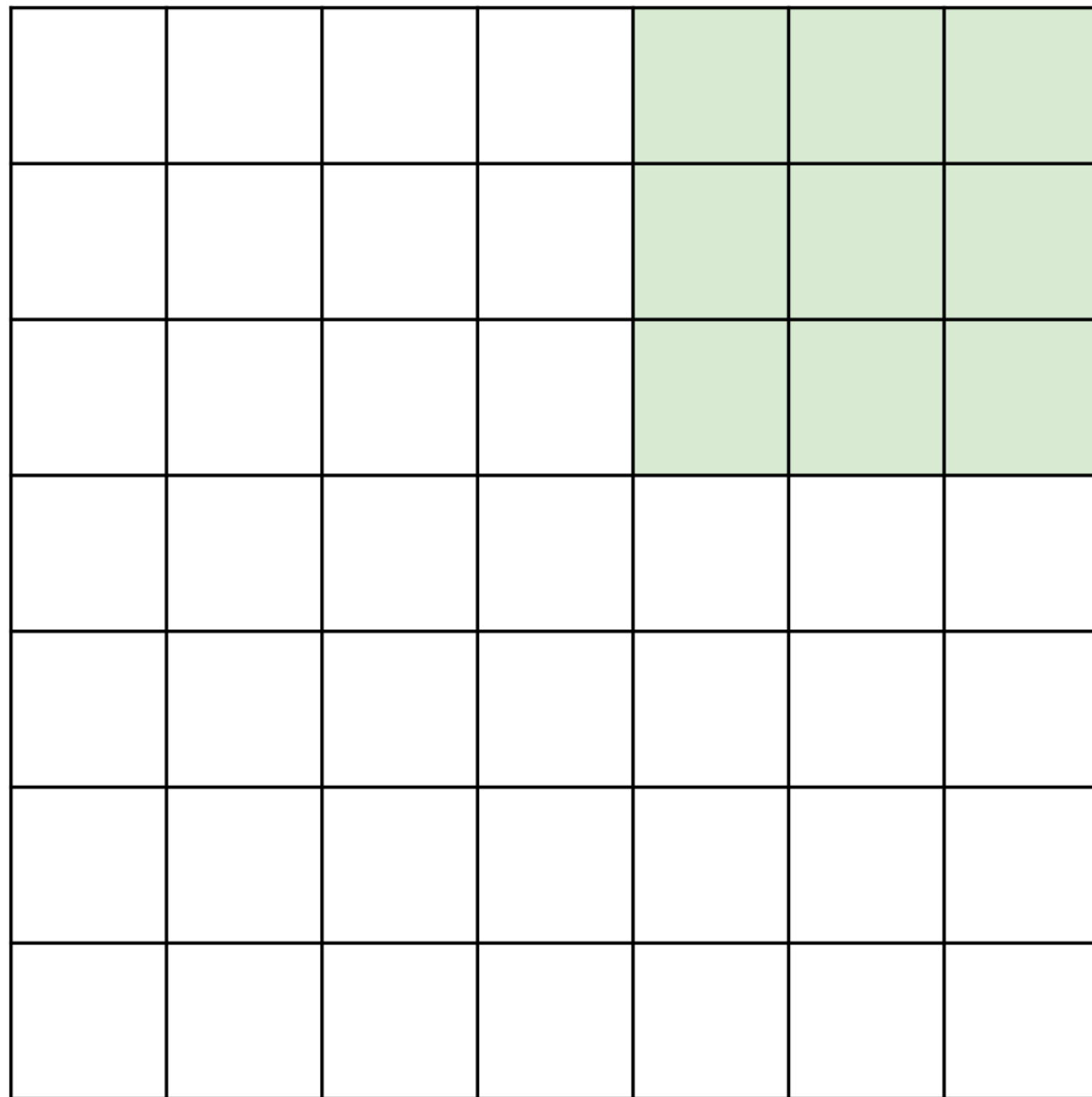
Filter: 3x3

Stride: 2

Output: 3x3



# Strided Convolution



Input: 7x7

Filter: 3x3

Output: 3x3

Stride: 2

In general:

Input:  $W$

Filter:  $K$

Padding:  $P$

Stride:  $S$

Output:  $(W - K + 2P) / S + 1$





# Dilated Convolution

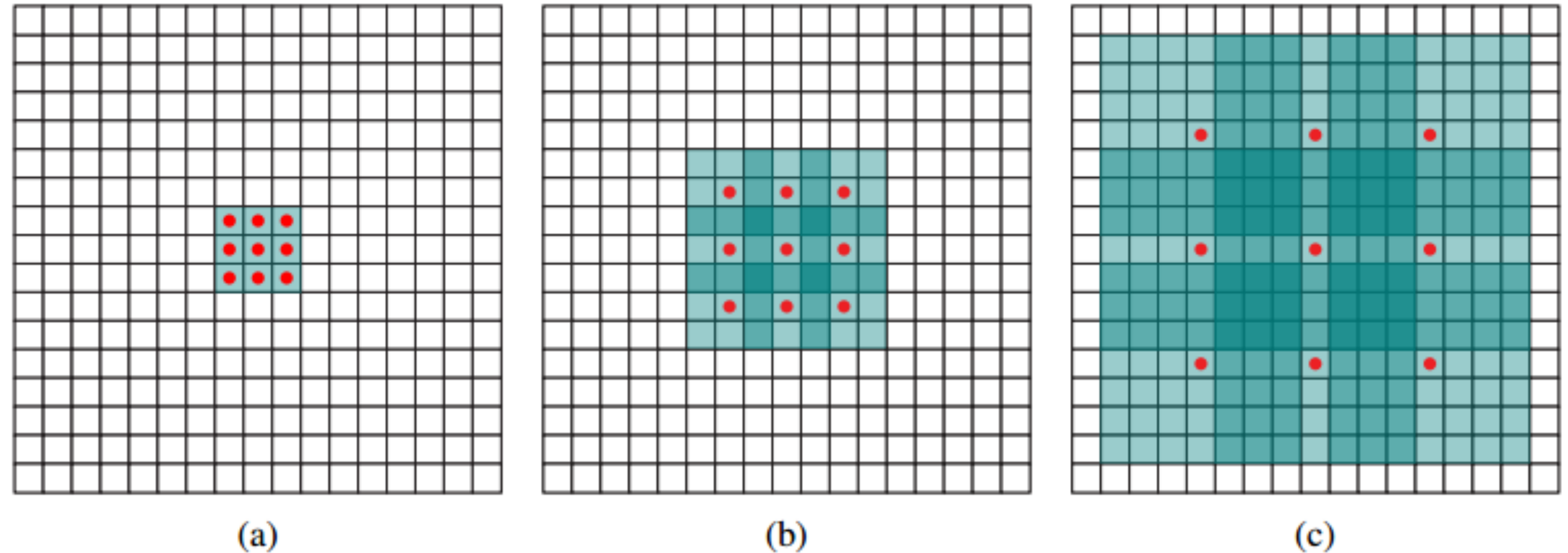
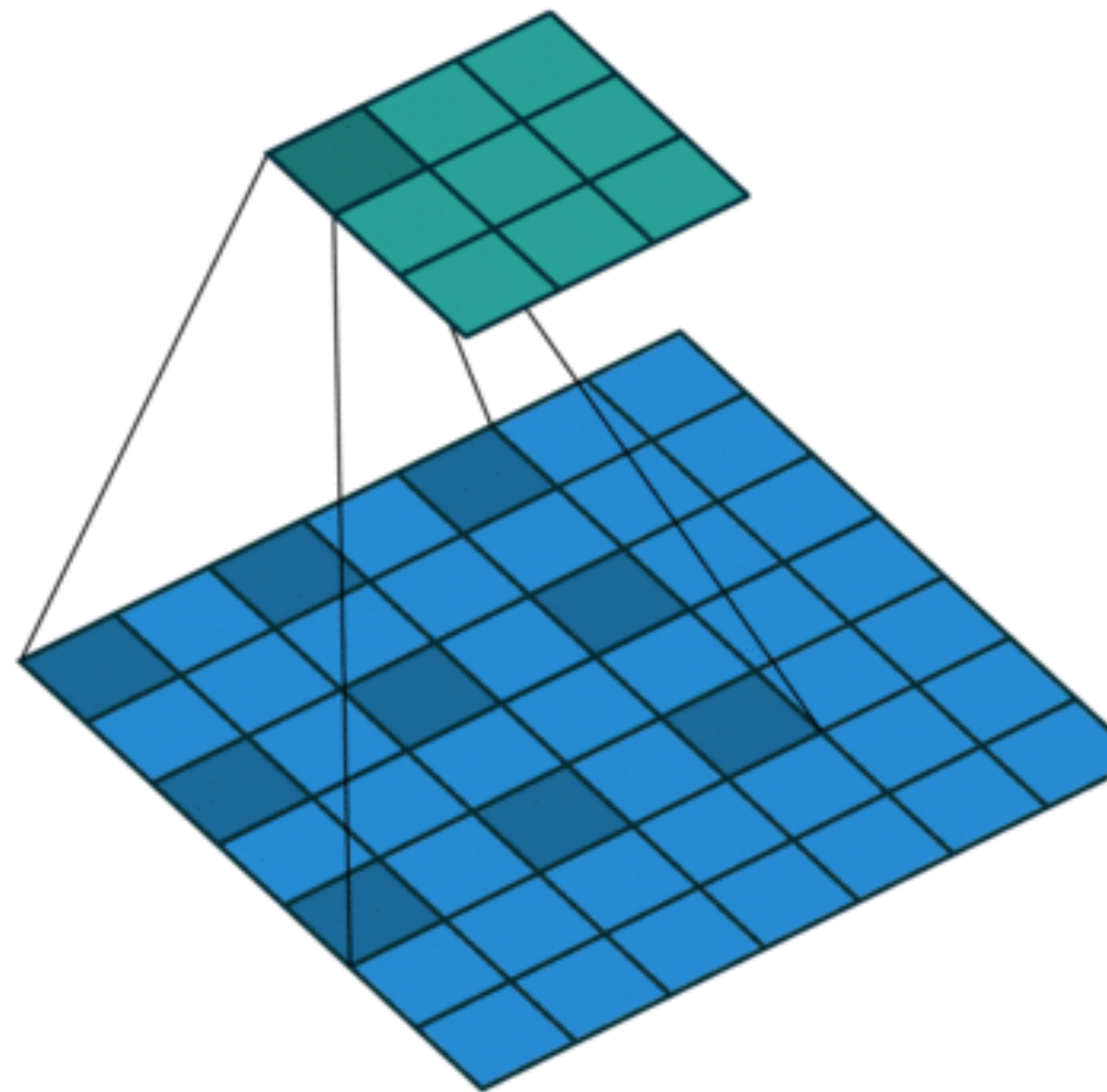


Figure 1: Systematic dilation supports exponential expansion of the receptive field without loss of resolution or coverage. (a)  $F_1$  is produced from  $F_0$  by a 1-dilated convolution; each element in  $F_1$  has a receptive field of  $3 \times 3$ . (b)  $F_2$  is produced from  $F_1$  by a 2-dilated convolution; each element in  $F_2$  has a receptive field of  $7 \times 7$ . (c)  $F_3$  is produced from  $F_2$  by a 4-dilated convolution; each element in  $F_3$  has a receptive field of  $15 \times 15$ . The number of parameters associated with each layer is identical. The receptive field grows exponentially while the number of parameters grows linearly.

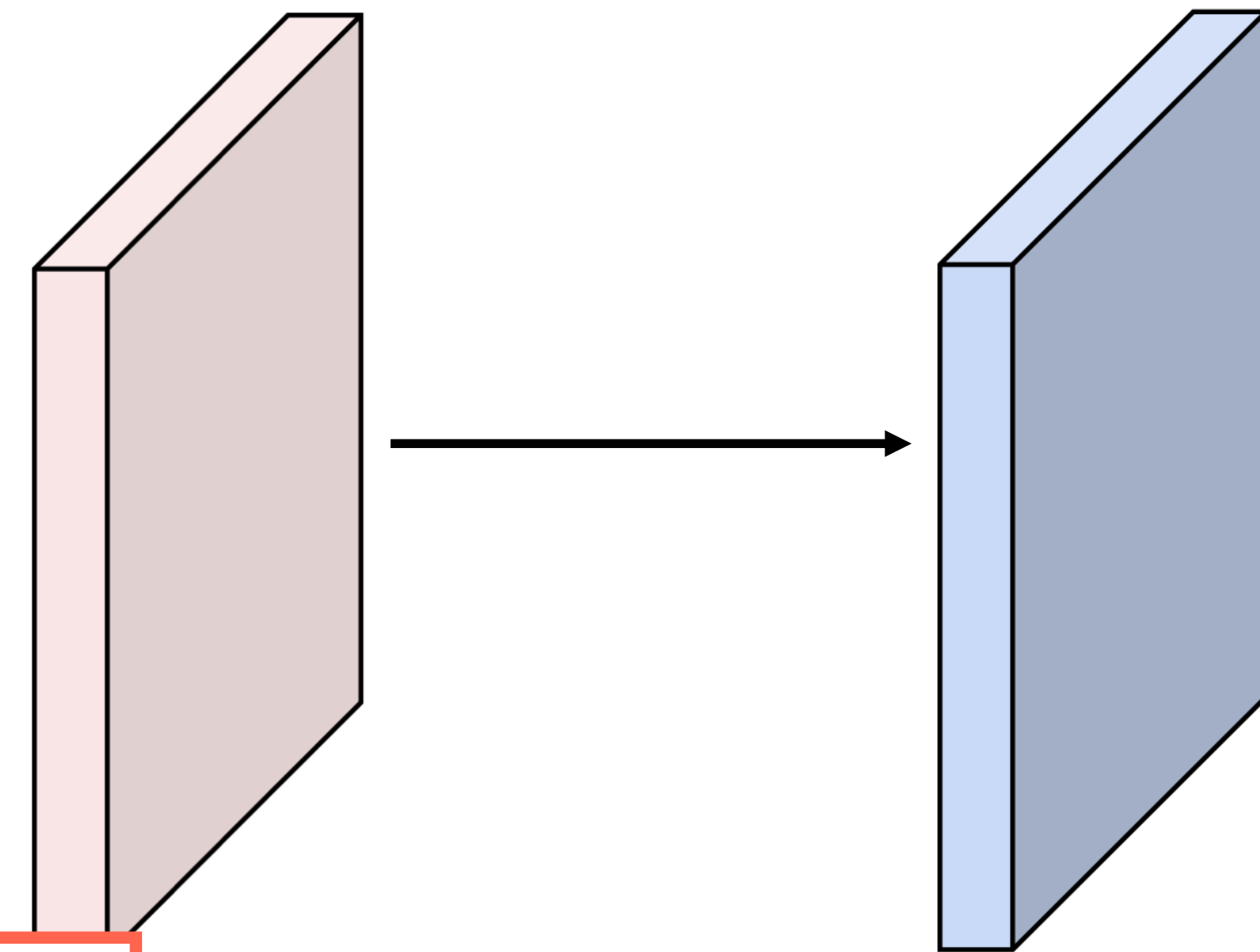


# Convolution Example

Input volume:  $3 \times 32 \times 32$

10  $5 \times 5$  filters with stride 1, pad 2

**Q:** What is the output volume size?



Input:  $W$   
Filter:  $K$   
Padding:  $P$   
Stride:  $S$   
Output:  $(W - K + 2P) / S + 1$



# Convolution Example

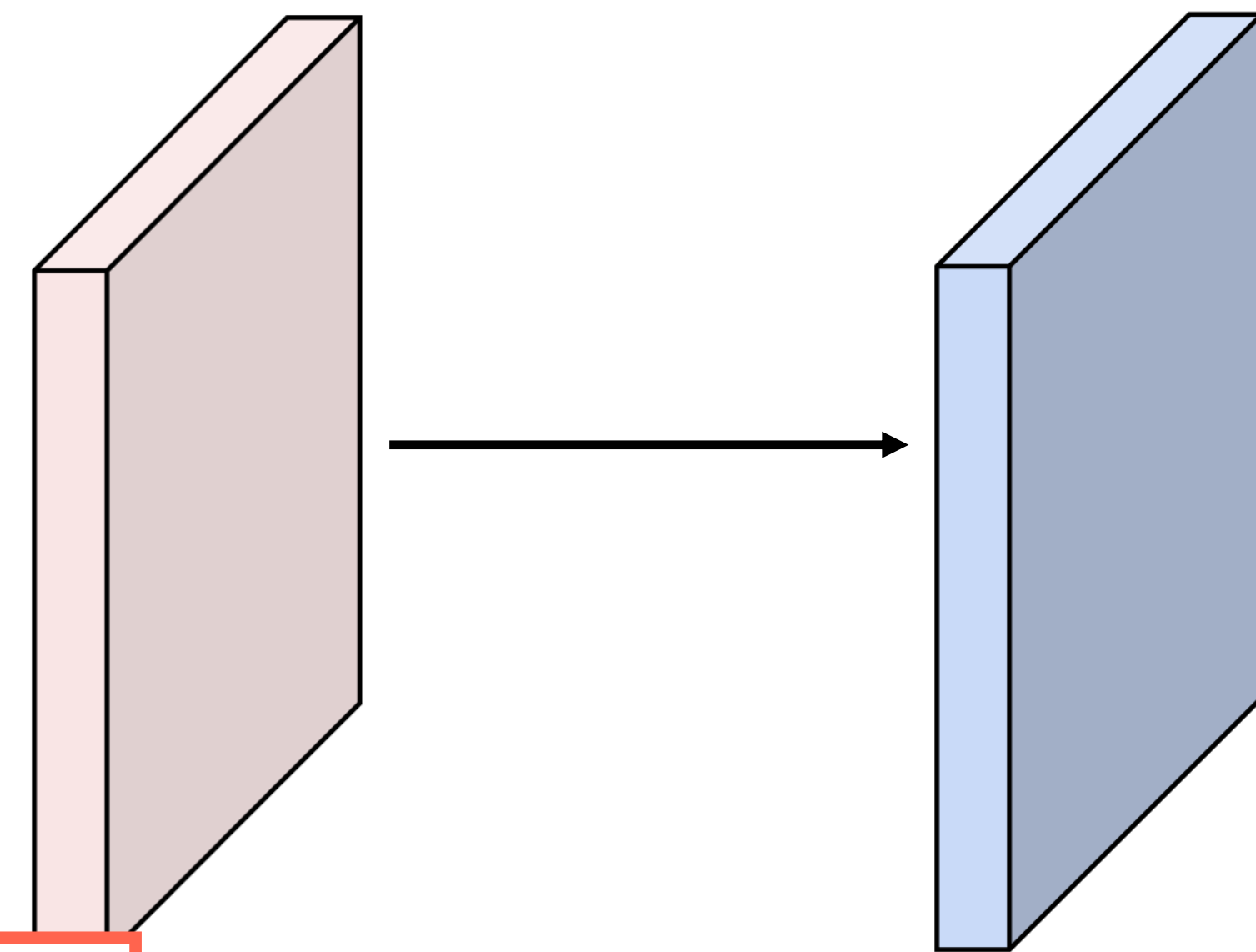
Input volume: 3 x 32 x 32

10 5x5 filters with stride 1, pad 2

**Q:** What is the output volume size?

$$(32 - 5 + 2 * 2) / 1 + 1 = 32 \text{ spatially}$$

So, 10 x 32 x 32 output



Input: W  
Filter: K  
Padding: P  
Stride: S  
Output:  $(W - K + 2P) / S + 1$



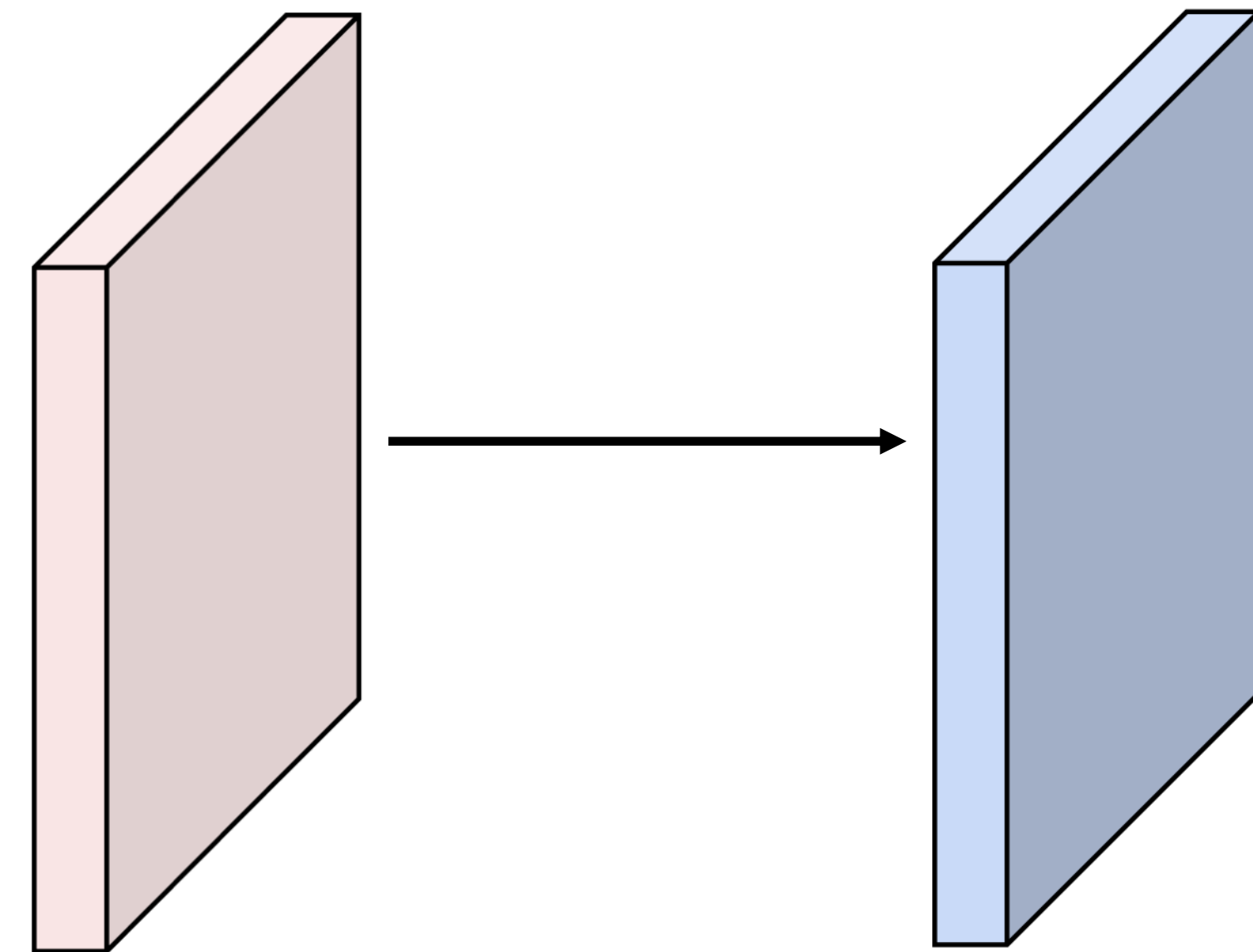
# Convolution Example

Input volume:  $3 \times 32 \times 32$

10  $5 \times 5$  filters with stride 1, pad 2

Output volume size:  $10 \times 32 \times 32$

**Q:** What is the number of learnable parameters?





# Convolution Example

Input volume: 3 x 32 x 32

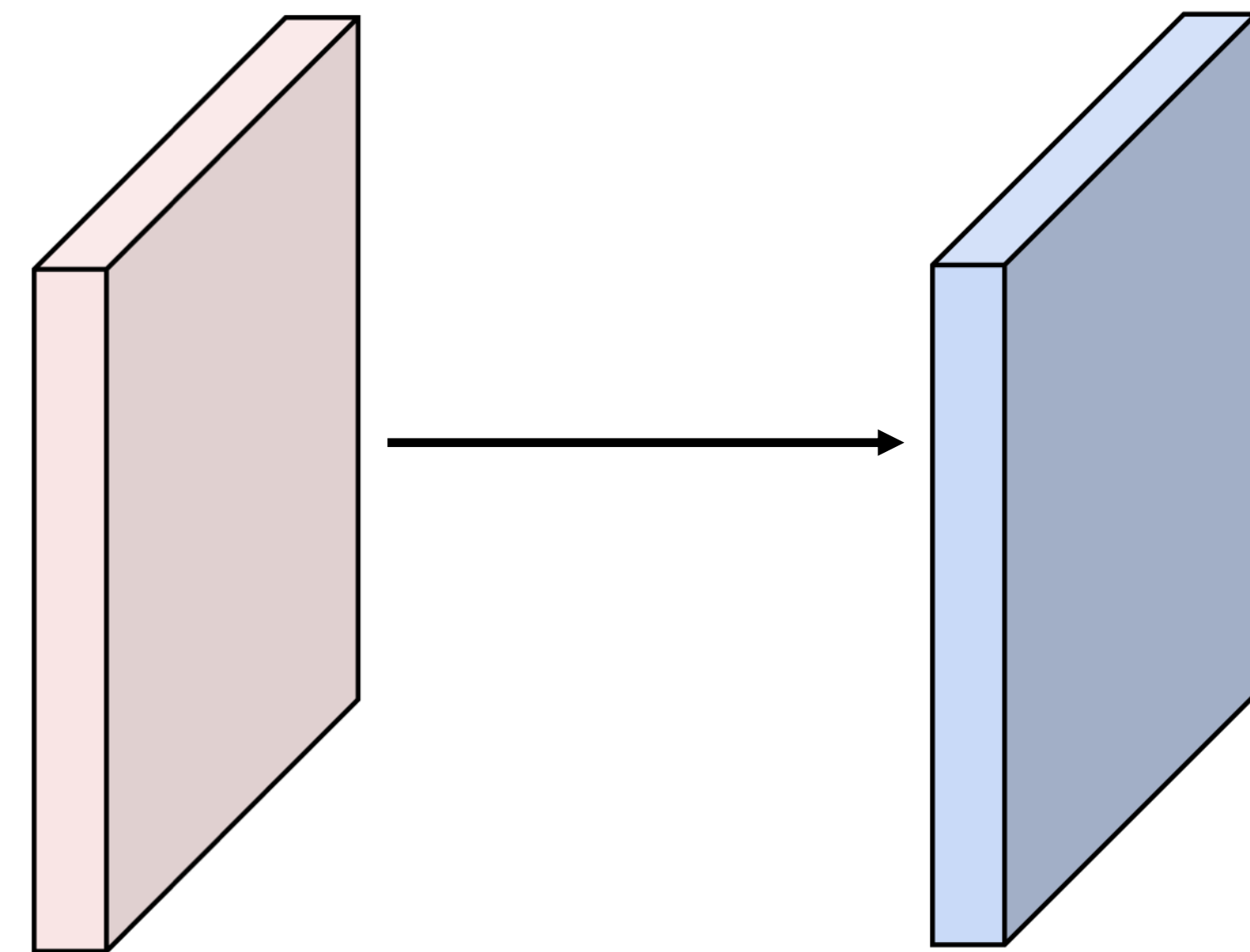
10 5x5 filters with stride 1, pad 2

Output volume size: 10 x 32 x 32

**Q:** What is the number of learnable parameters?

Parameters per filter:  $(3 \cdot 5 \cdot 5) + 1 = 76$

10 filters, so total is  $10 \cdot 76 = 760$





# Convolution Example

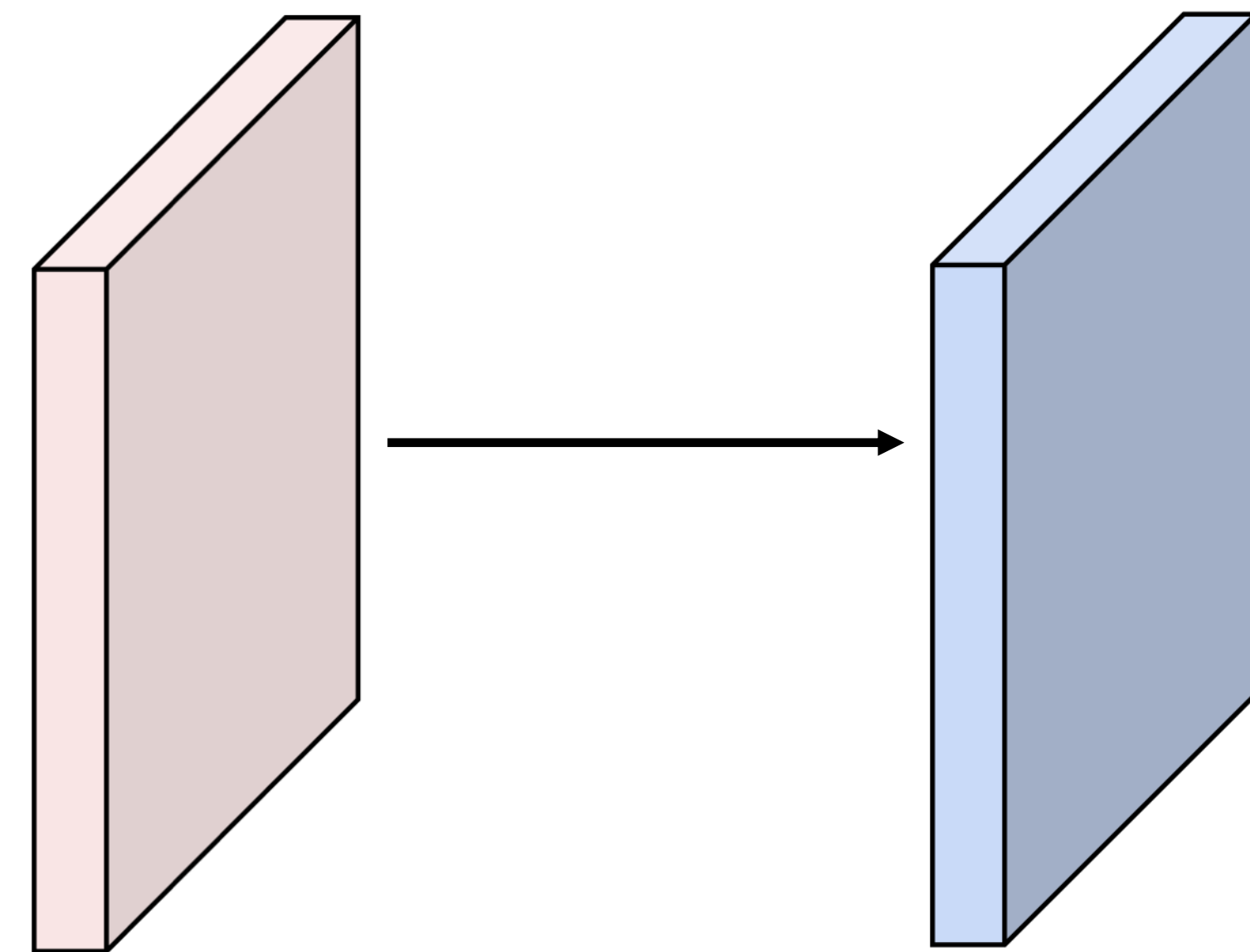
Input volume:  $3 \times 32 \times 32$

10  $5 \times 5$  filters with stride 1, pad 2

Output volume size:  $10 \times 32 \times 32$

Number of learnable parameters: 760

**Q:** What is the number of multiply-add operations?





# Convolution Example

Input volume:  $3 \times 32 \times 32$

10  $5 \times 5$  filters with stride 1, pad 2

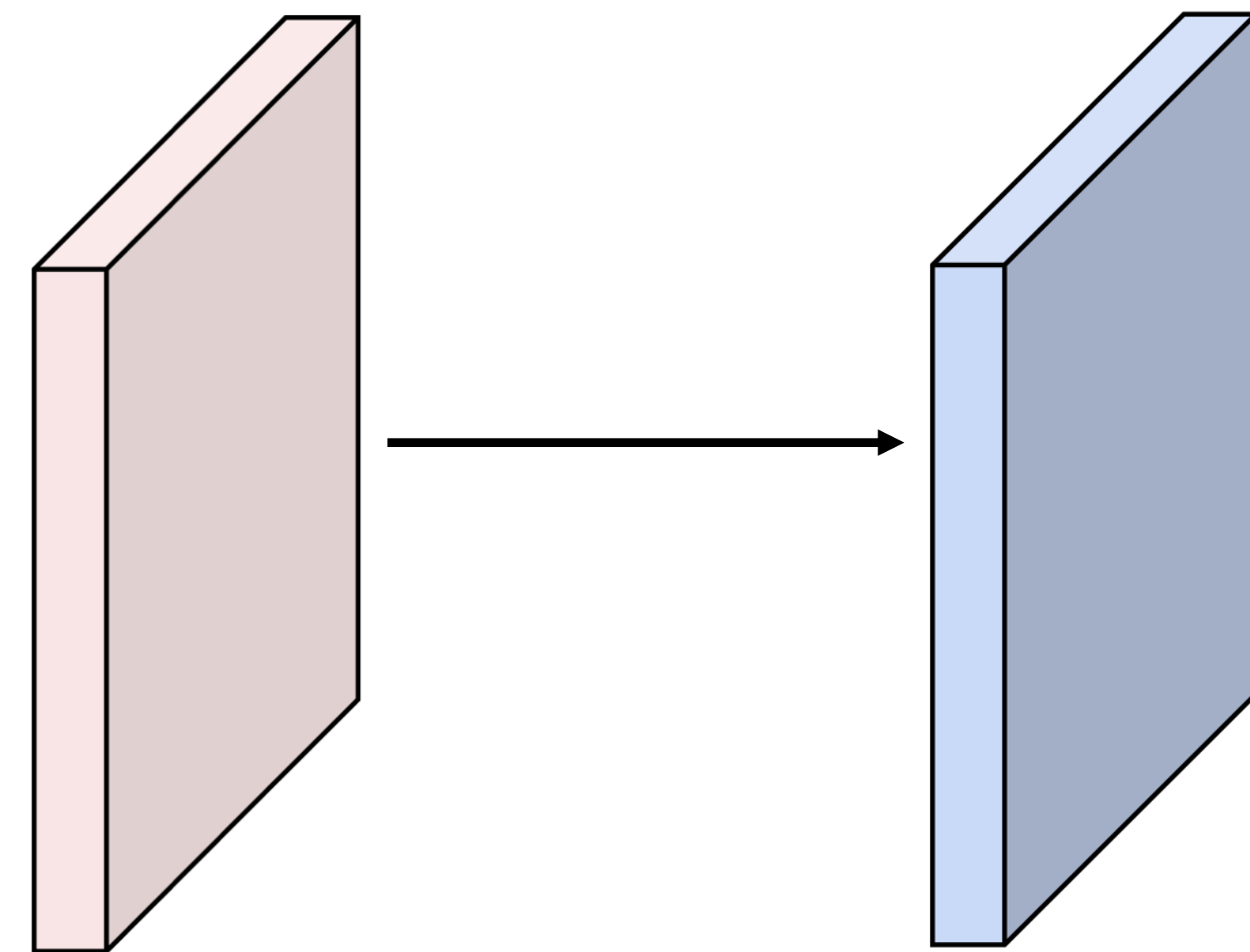
Output volume size:  $10 \times 32 \times 32$

Number of learnable parameters: 760

**Q:** What is the number of multiply-add operations?

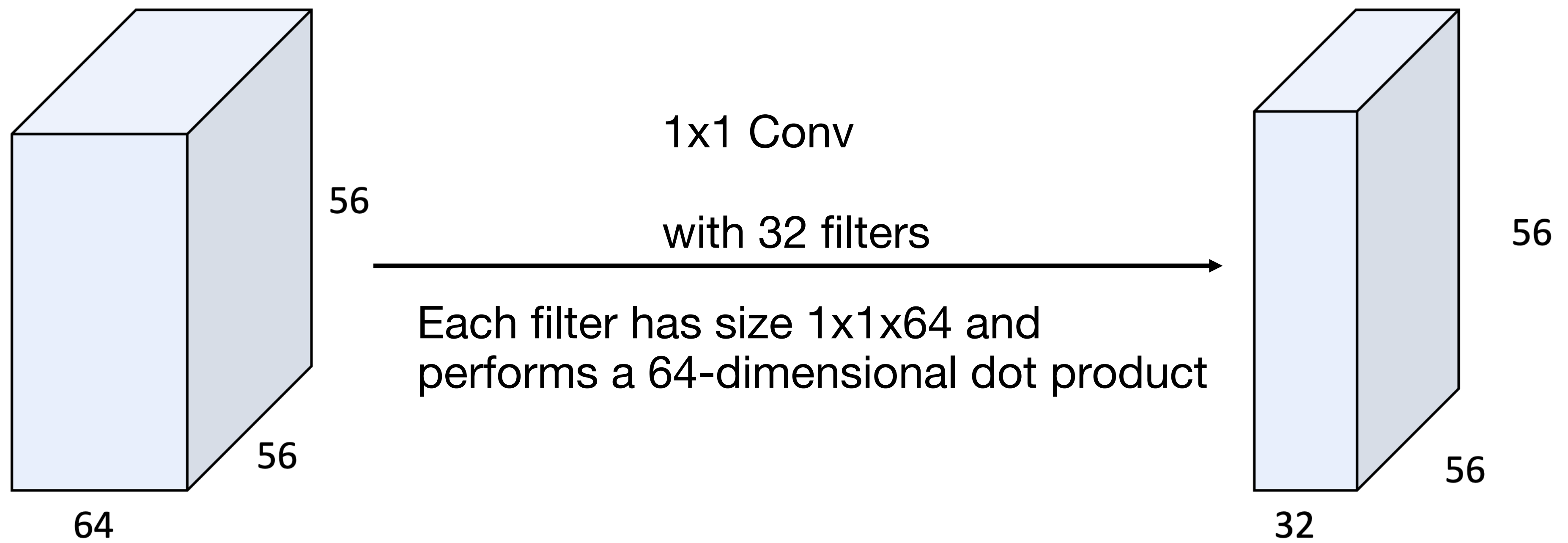
$10 \times 32 \times 32 = 10,240$  outputs, each from inner product

of two  $3 \times 5 \times 5$  tensors, so total =  $75 * 10,240 = \underline{\underline{768,000}}$





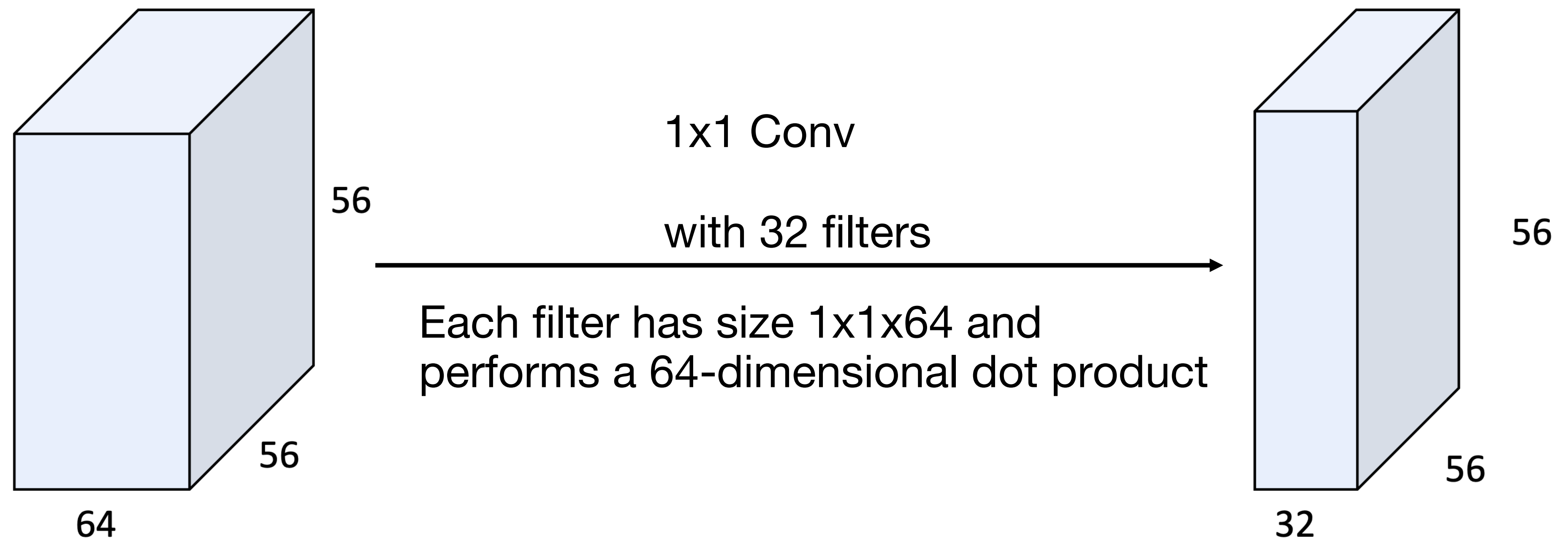
# Example: 1x1 Convolution







# Example: 1x1 Convolution



Stacking 1x1 conv layers gives MLP operating on each input position



# Convolution Summary

---

**Input:**  $C_{in} \times H \times W$

**Hyperparameters:**

- **Kernel size:**  $K_H \times K_W$
- **Number filters:**  $C_{out}$
- **Padding:**  $P$
- **Stride:**  $S$

**Weight matrix:**  $C_{out} \times C_{in} \times K_H \times K_W$

giving  $C_{out}$  filters of size  $C_{in} \times K_H \times K_W$

**Bias vector:**  $C_{out}$

**Output size:**  $C_{out} \times H' \times W'$  where:

- $H' = (H - K + 2P) / S + 1$
- $W' = (W - K + 2P) / S + 1$



# Convolution Summary

**Input:**  $C_{in} \times H \times W$

**Hyperparameters:**

- **Kernel size:**  $K_H \times K_W$
- **Number filters:**  $C_{out}$
- **Padding:**  $P$
- **Stride:**  $S$

**Weight matrix:**  $C_{out} \times C_{in} \times K_H \times K_W$   
giving  $C_{out}$  filters of size  $C_{in} \times K_H \times K_W$

**Bias vector:**  $C_{out}$

**Output size:**  $C_{out} \times H' \times W'$  where:

- $H' = (H - K + 2P) / S + 1$
- $W' = (W - K + 2P) / S + 1$

Common settings:

$K_H = K_W$  (Small square filters)

$P = (K - 1) / 2$  ("Same" padding)

$C_{in}, C_{out} = 32, 64, 128, 256$  (powers of 2)

$K = 3, P = 1, S = 1$  (3x3 conv)

$K = 5, P = 2, S = 1$  (5x5 conv)

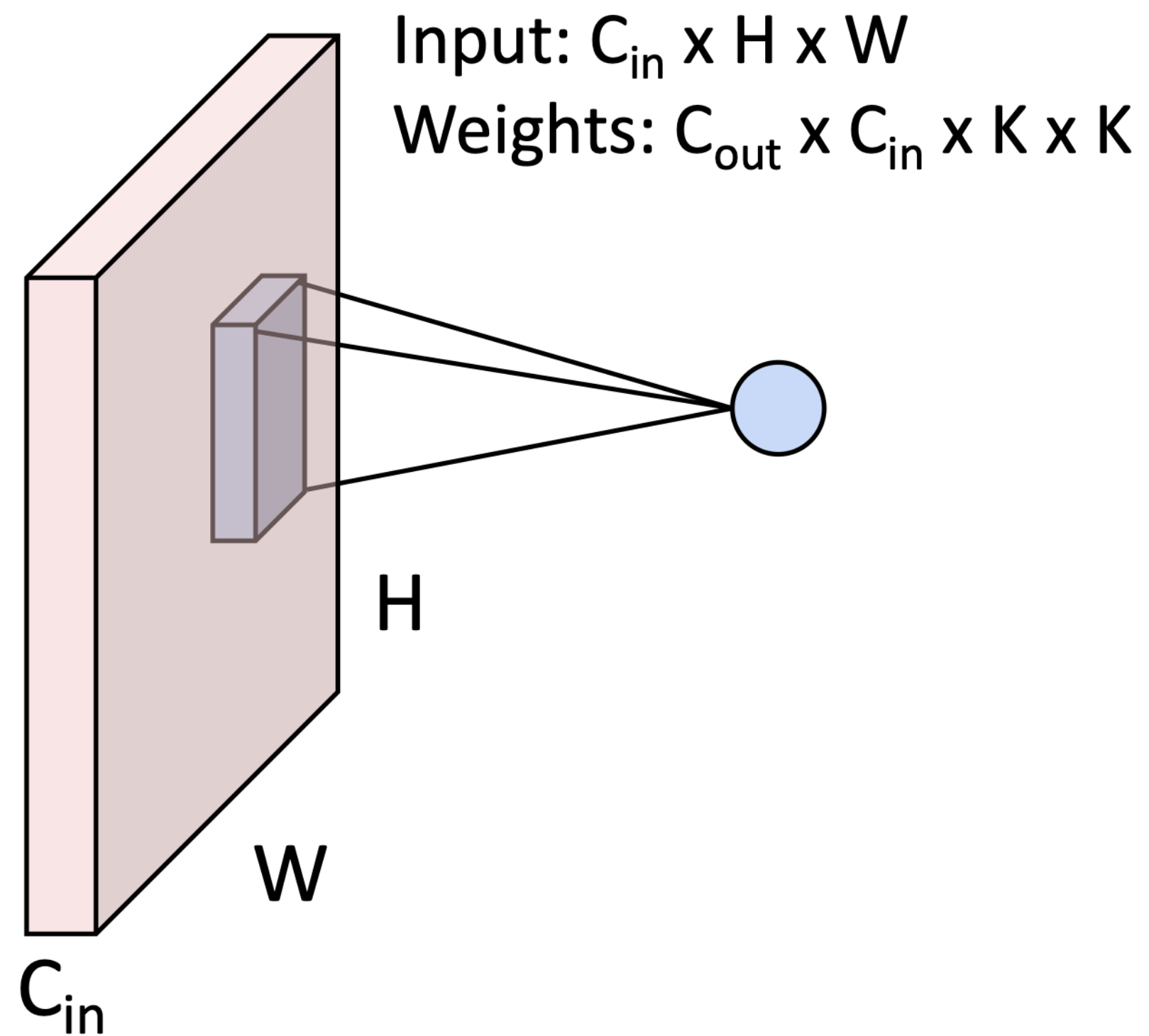
$K = 1, P = 0, S = 1$  (1x1 conv)

$K = 3, P = 1, S = 2$  (Downsample by 2)



# Other types of convolution

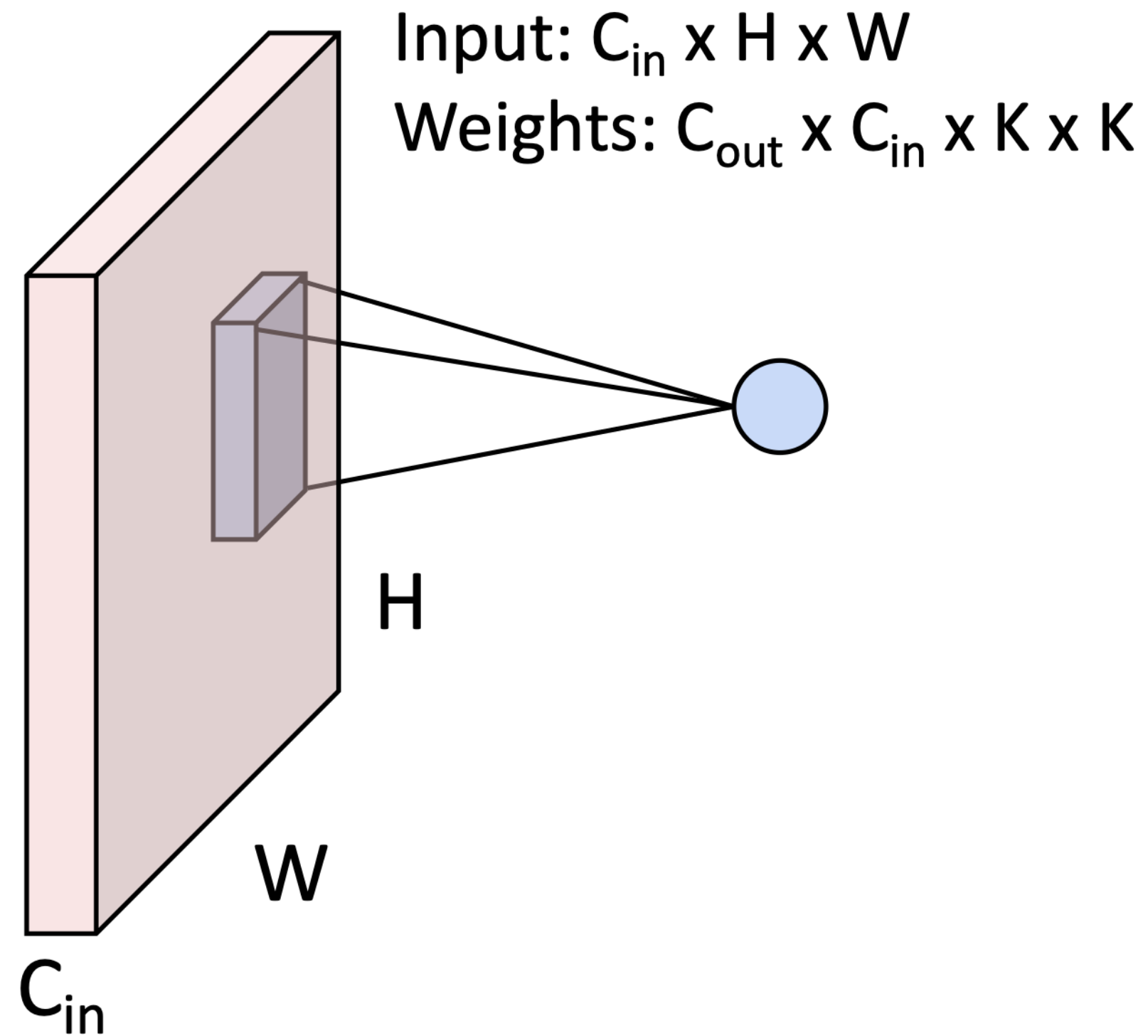
So far: 2D Convolution



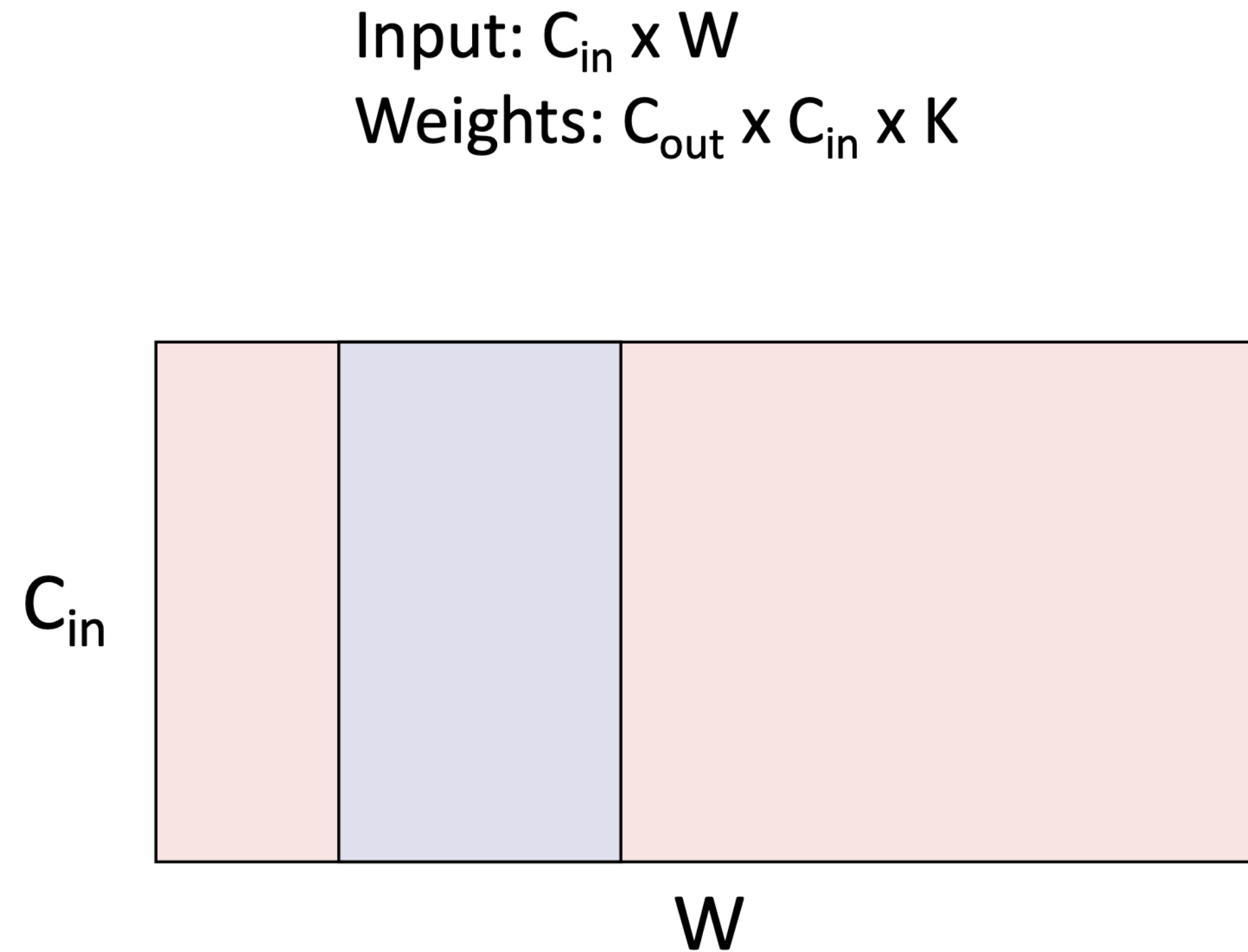


# Other types of convolution

So far: 2D Convolution



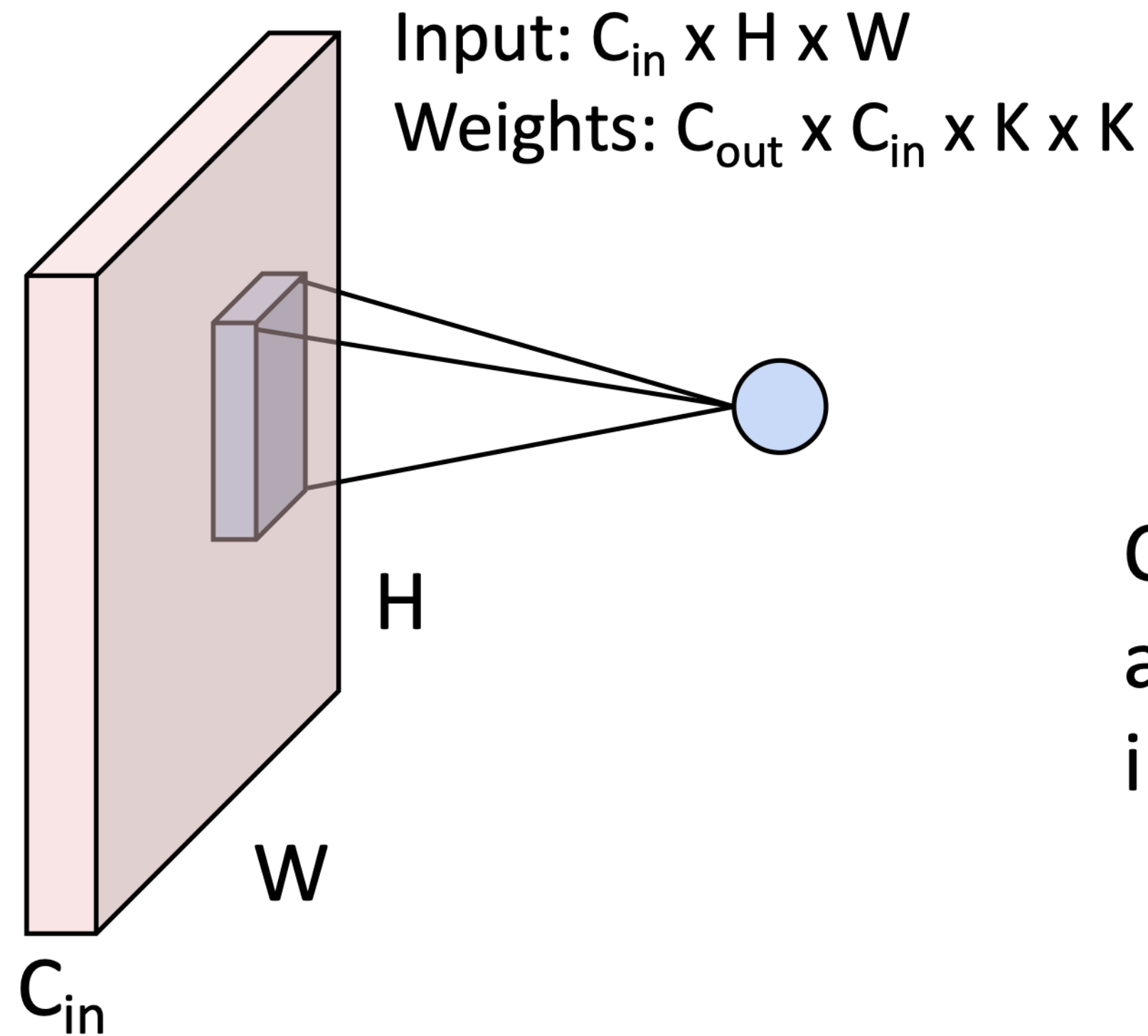
1D Convolution





# Other types of convolution

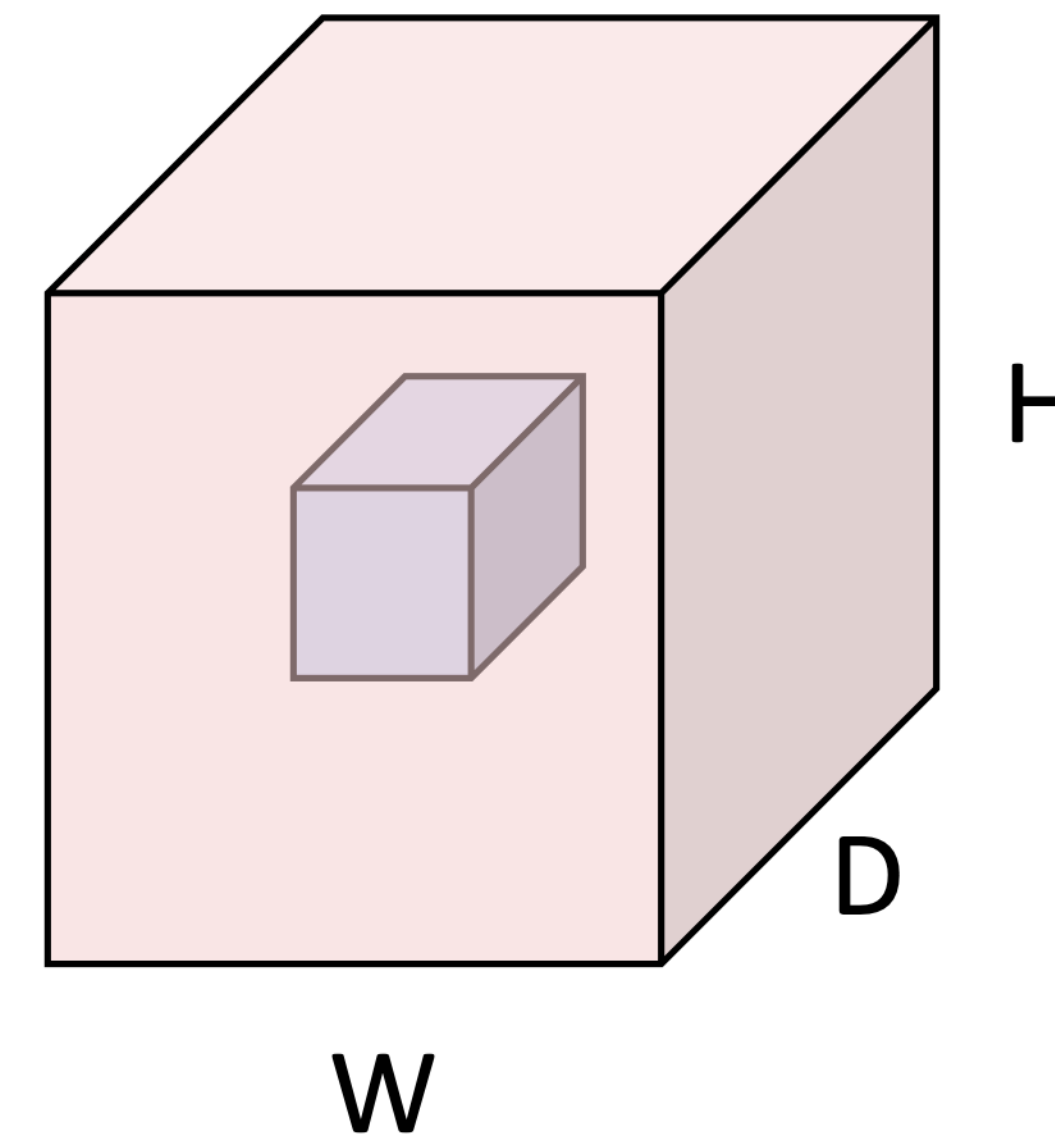
So far: 2D Convolution



3D Convolution

Input:  $C_{in} \times H \times W \times D$   
Weights:  $C_{out} \times C_{in} \times K \times K \times K$

$C_{in}$ -dim vector  
at each point  
in the volume





# PyTorch Convolution Layer

## Conv2d

```
CLASS torch.nn.Conv2d(in_channels, out_channels, kernel_size, stride=1, padding=0, dilation=1, groups=1, bias=True, padding_mode='zeros')
```

[\[SOURCE\]](#)

Applies a 2D convolution over an input signal composed of several input planes.

In the simplest case, the output value of the layer with input size  $(N, C_{in}, H, W)$  and output  $(N, C_{out}, H_{out}, W_{out})$  can be precisely described as:

$$\text{out}(N_i, C_{out_j}) = \text{bias}(C_{out_j}) + \sum_{k=0}^{C_{in}-1} \text{weight}(C_{out_j}, k) \star \text{input}(N_i, k)$$



# PyTorch Convolution Layer

---

## Conv2d

**CLASS** `torch.nn.Conv2d(in_channels, out_channels, kernel_size, stride=1, padding=0, dilation=1, groups=1, bias=True, padding_mode='zeros')`

[\[SOURCE\]](#)

## Conv1d

**CLASS** `torch.nn.Conv1d(in_channels, out_channels, kernel_size, stride=1, padding=0, dilation=1, groups=1, bias=True, padding_mode='zeros')`

[\[SOURCE\]](#) [🔗](#)

## Conv3d

**CLASS** `torch.nn.Conv3d(in_channels, out_channels, kernel_size, stride=1, padding=0, dilation=1, groups=1, bias=True, padding_mode='zeros')`

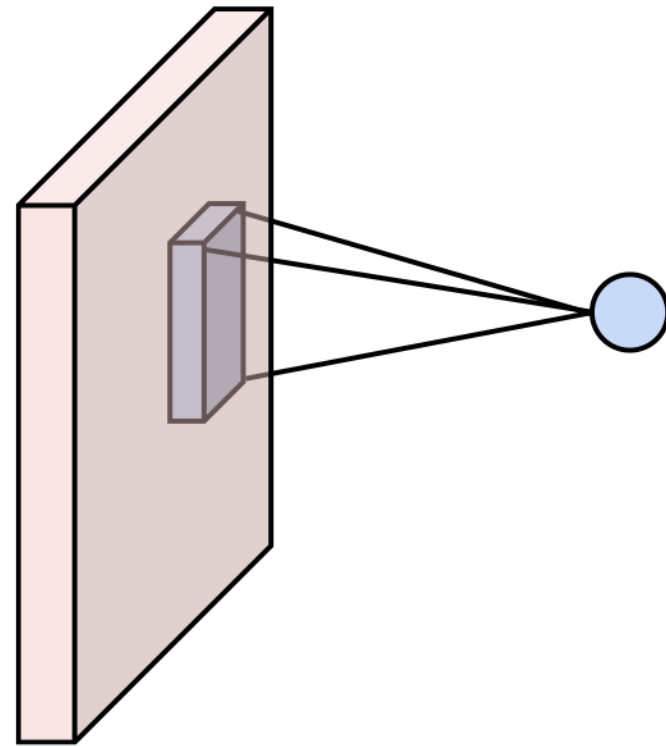
[\[SOURCE\]](#)



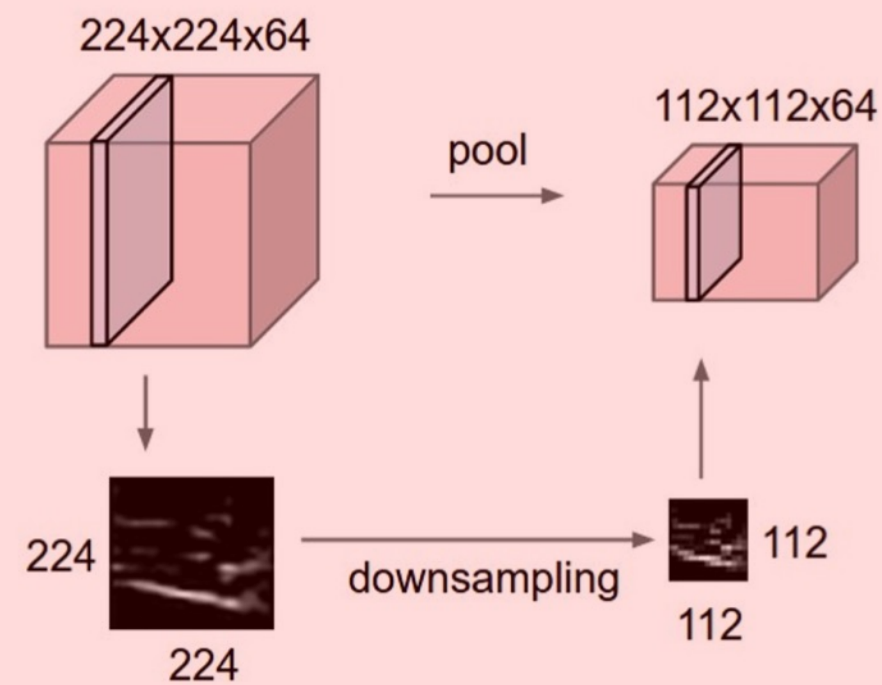


# Components of Convolutional Networks

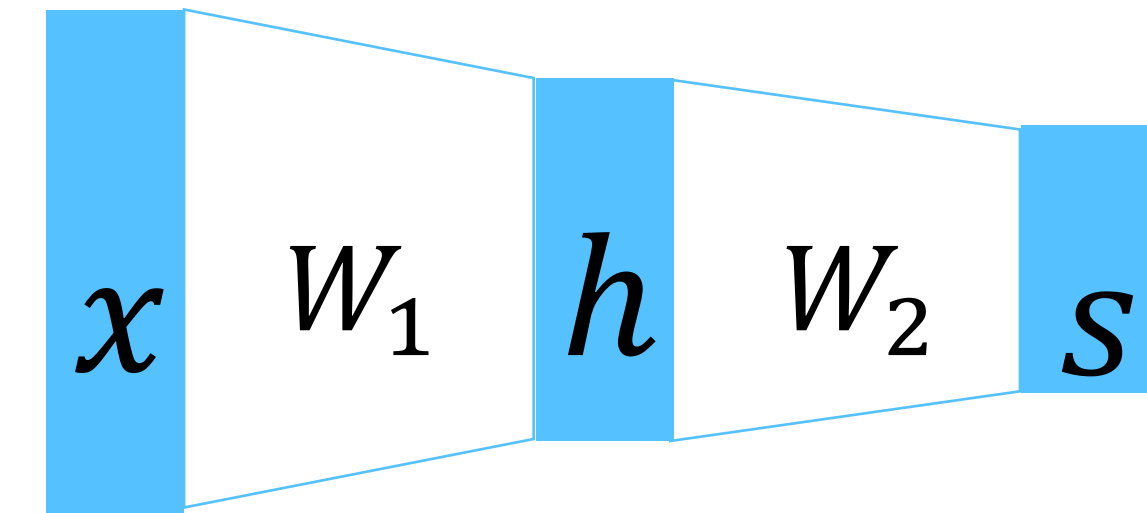
## Convolution Layers



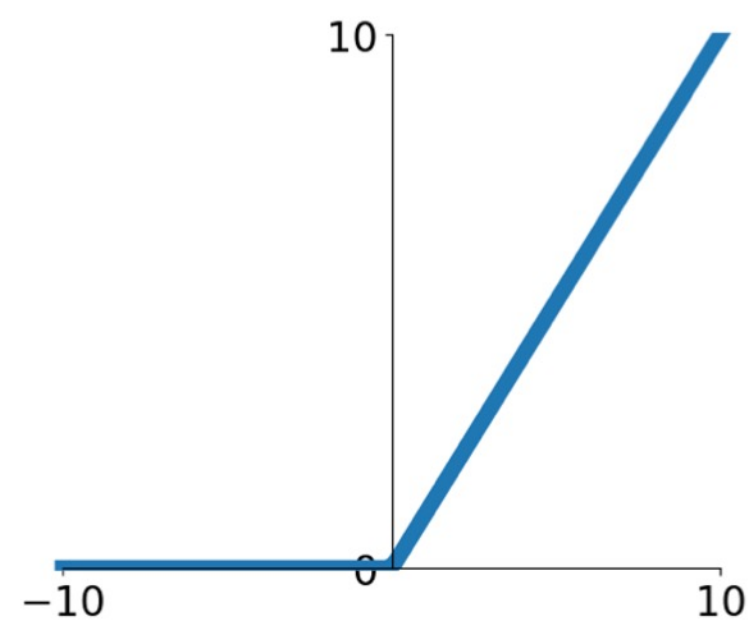
## Pooling Layers



## Fully-Connected Layers



## Activation Function

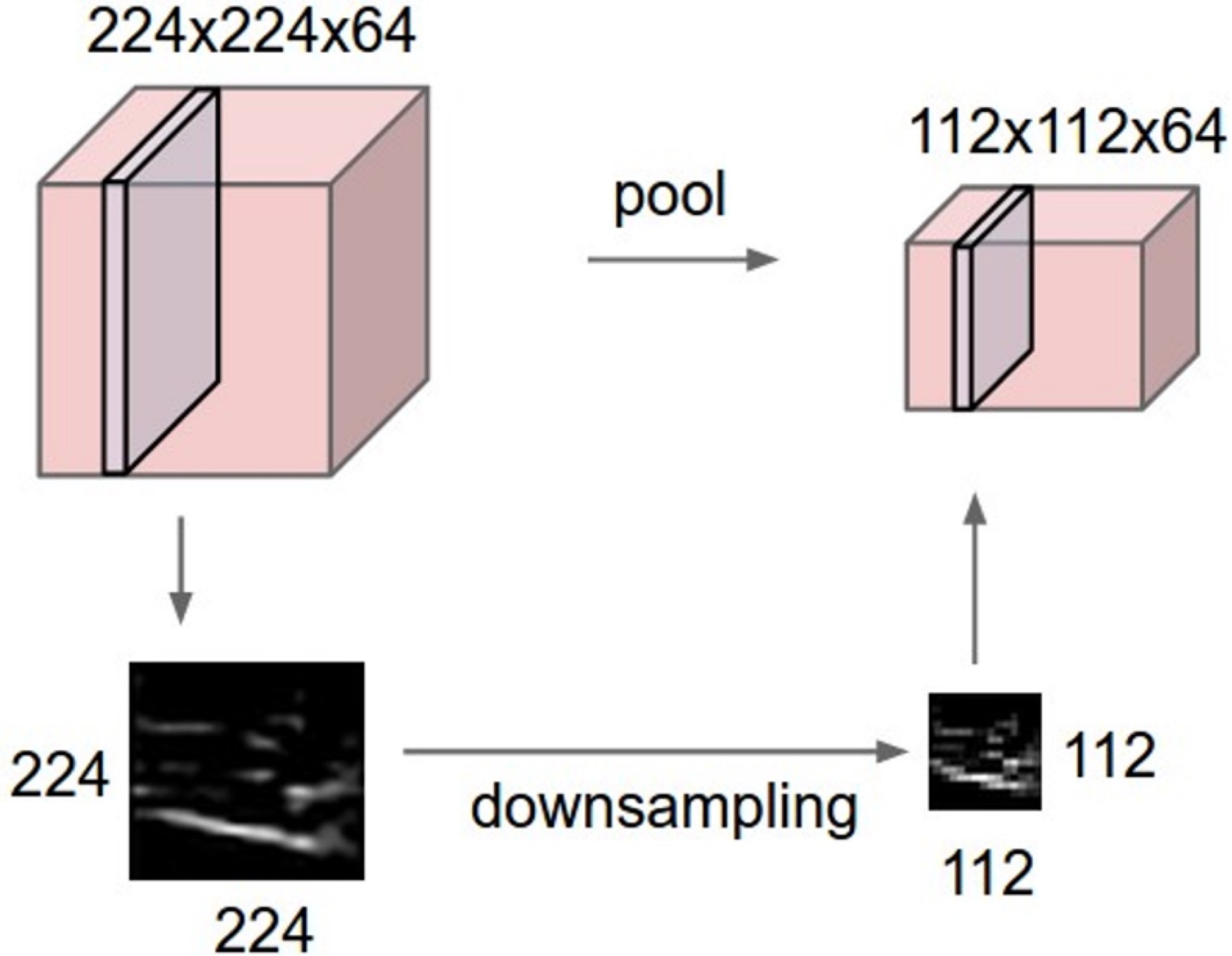


## Normalization

$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \epsilon}}$$



# Pooling Layers: Another way to downsample



**Hyperparameters:**

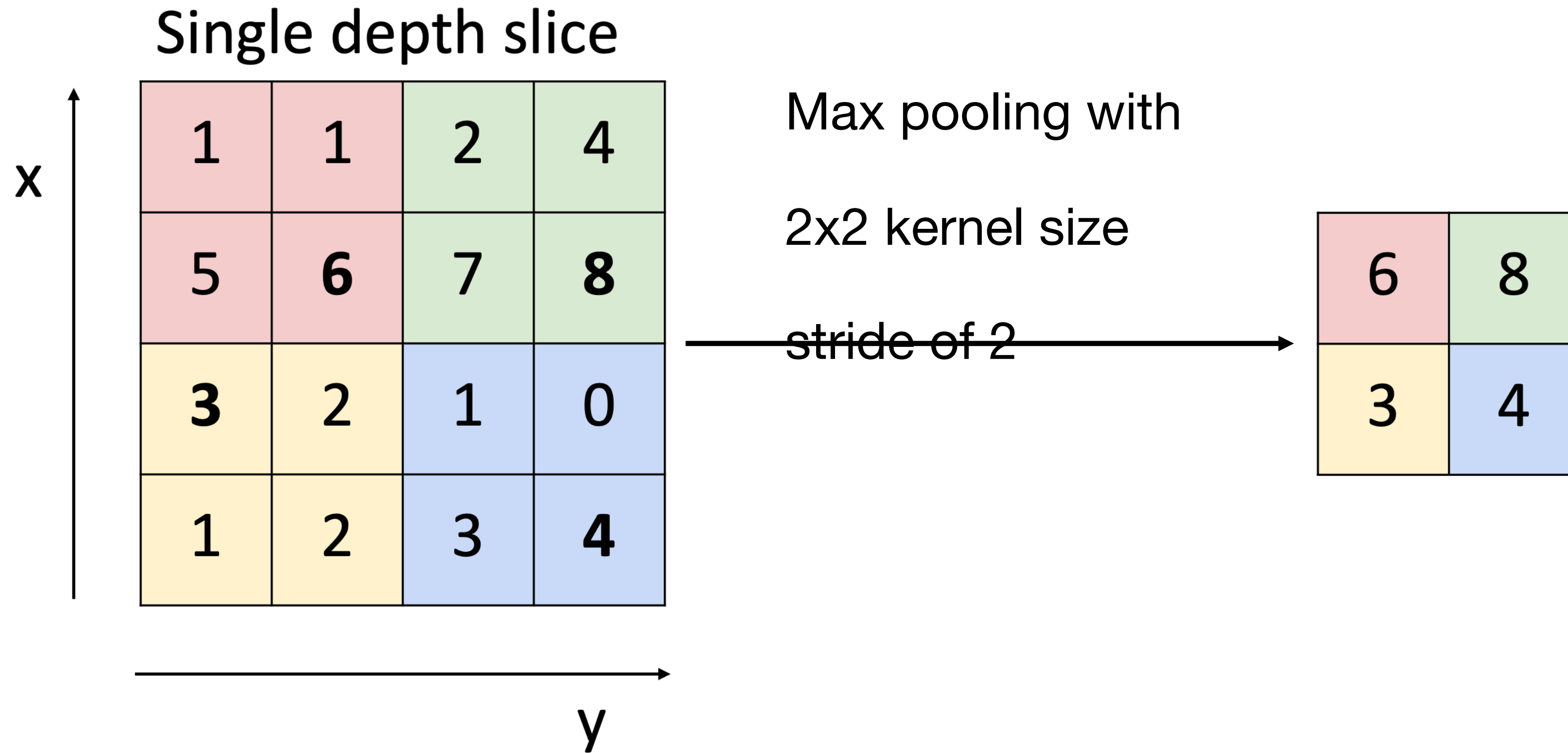
Kernel size

Stride

Pooling function

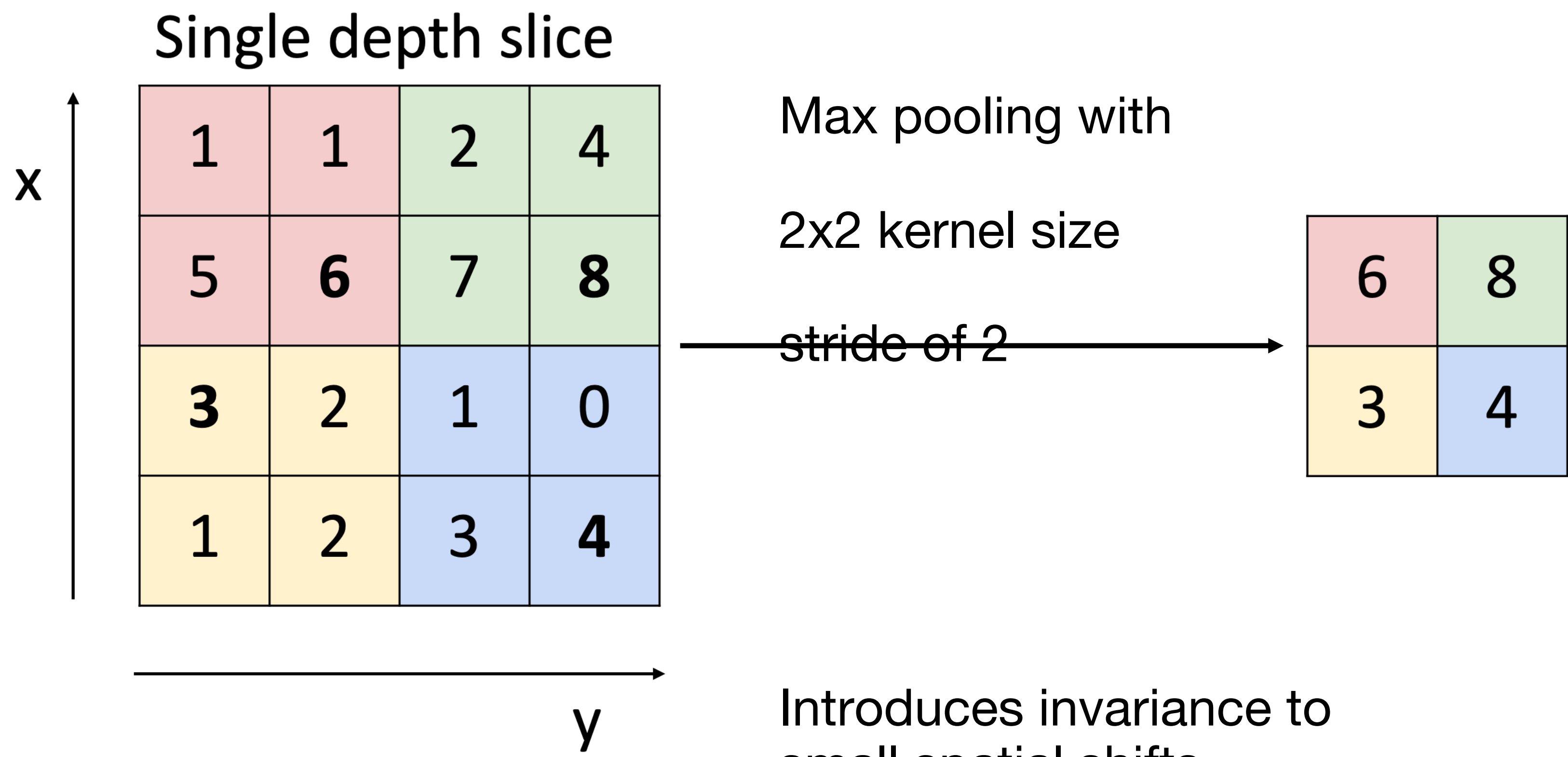


# Max Pooling





# Max Pooling



Introduces invariance to small spatial shifts

No learnable parameters!



# Pooling Summary

---

**Input:**  $C \times H \times W$

**Hyperparameters:**

- Kernel size:  $K$
- Stride:  $S$
- Pooling function (max, avg)

**Output:**  $C \times H' \times W'$  where

- $H' = (H - K) / S + 1$
- $W' = (W - K) / S + 1$

**Learnable parameters:** None!

Common settings:

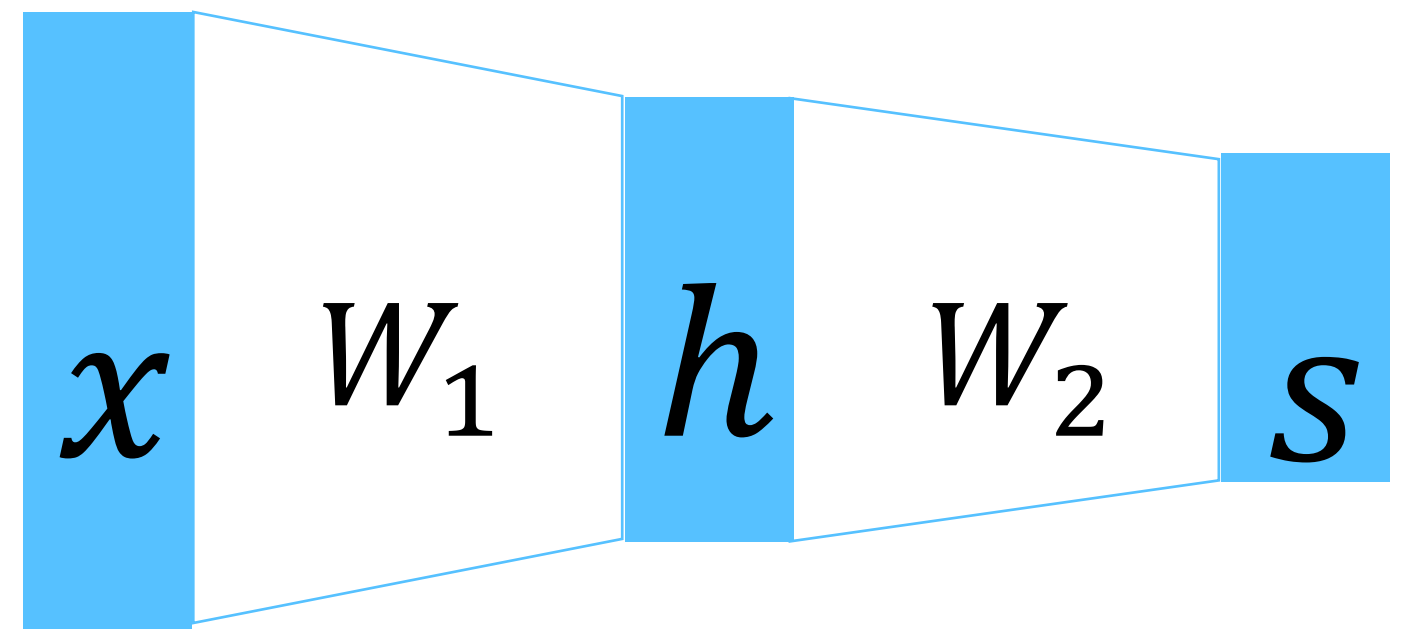
max,  $K = 2, S = 2$

max,  $K = 3, S = 2$  (AlexNet)

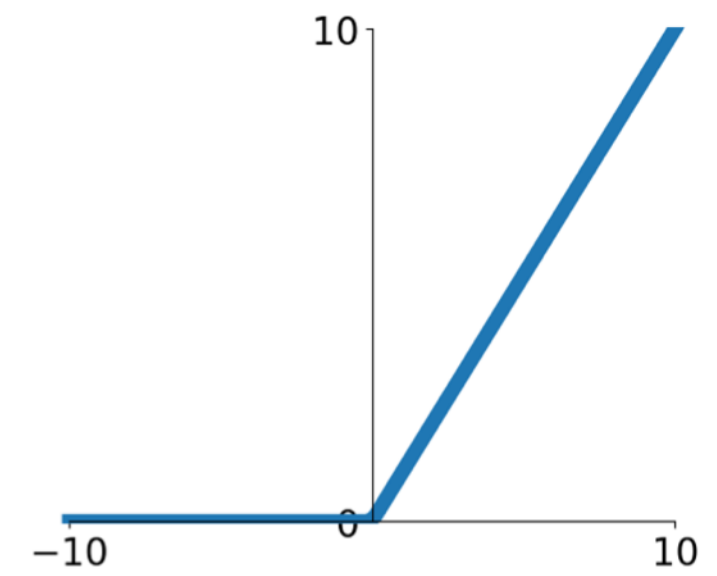


# Components of Convolutional Neural Networks

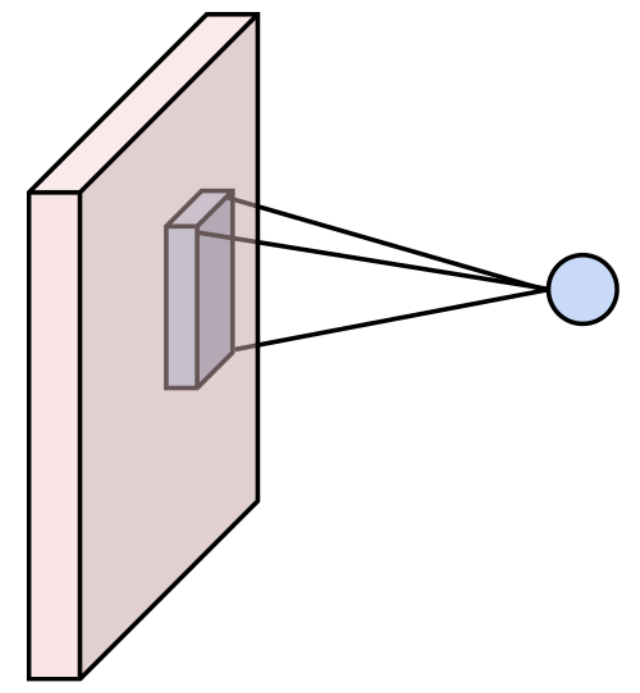
## Fully-Connected Layers



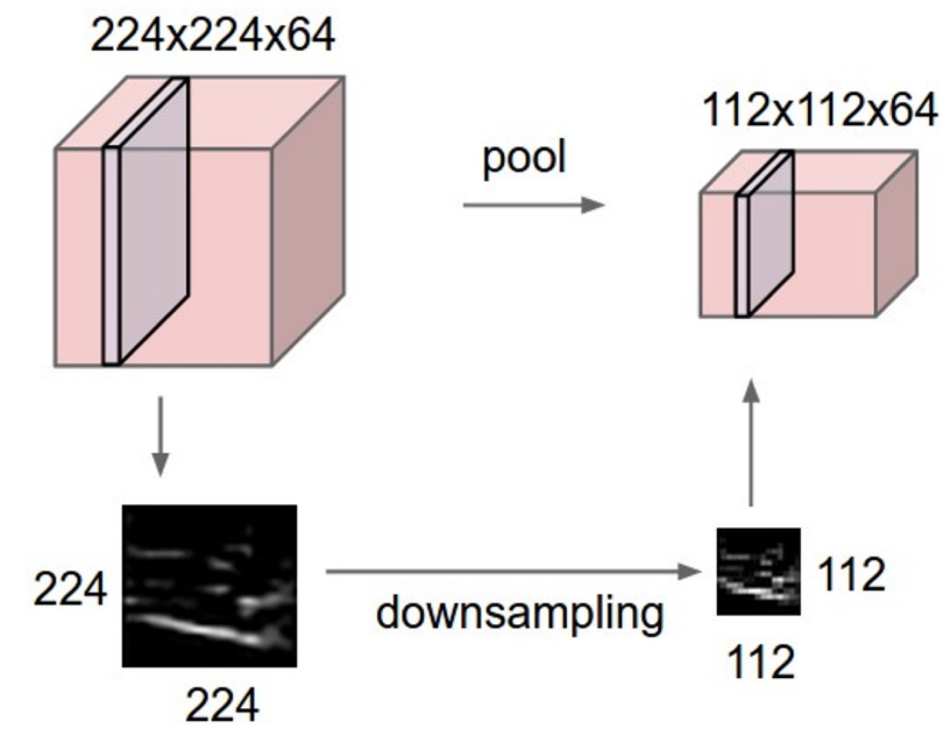
## Activation Functions



## Convolution Layers



## Pooling Layers



## Normalization

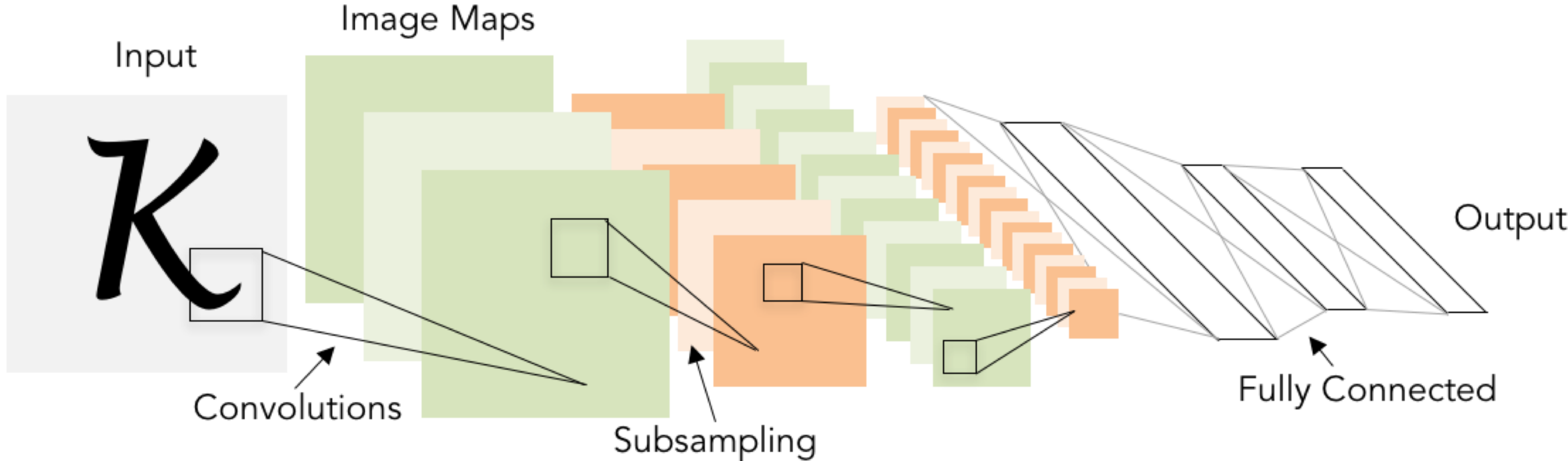
$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \epsilon}}$$



# Convolutional Neural Networks

Classic architecture: [Conv, ReLU, Pool] x N, flatten, [FC, ReLU] x N, FC

Example: LeNet-5

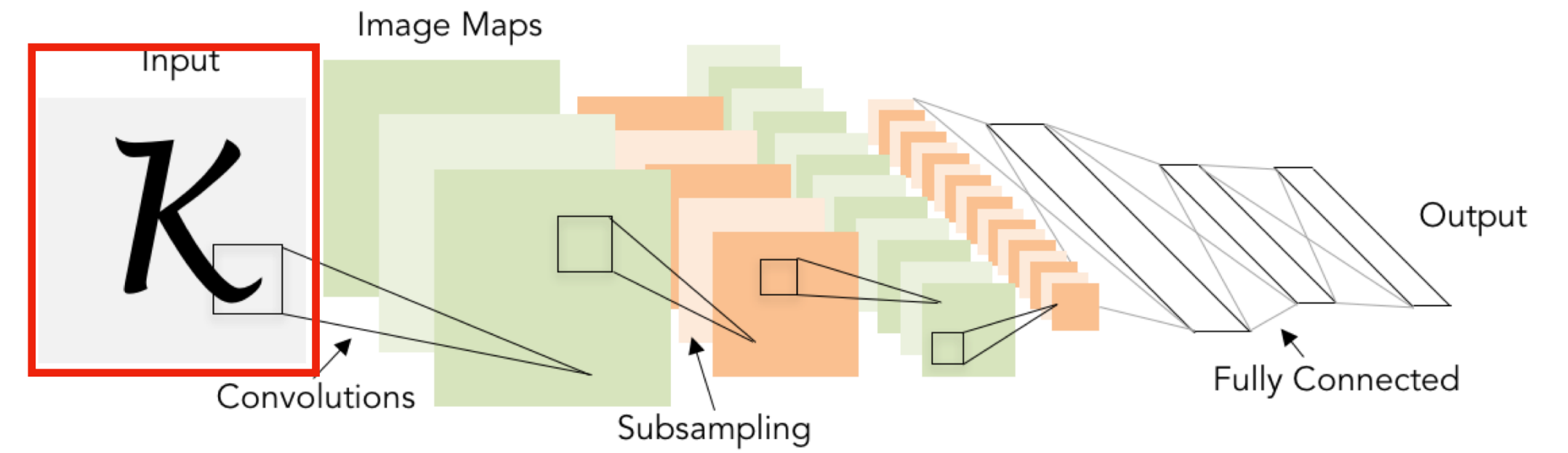


Lecun et al., "Gradient-based learning applied to document recognition", 1998



# Example: LeNet-5

Layer	Output Size	Weight Size
Input	1 x 28 x 28	

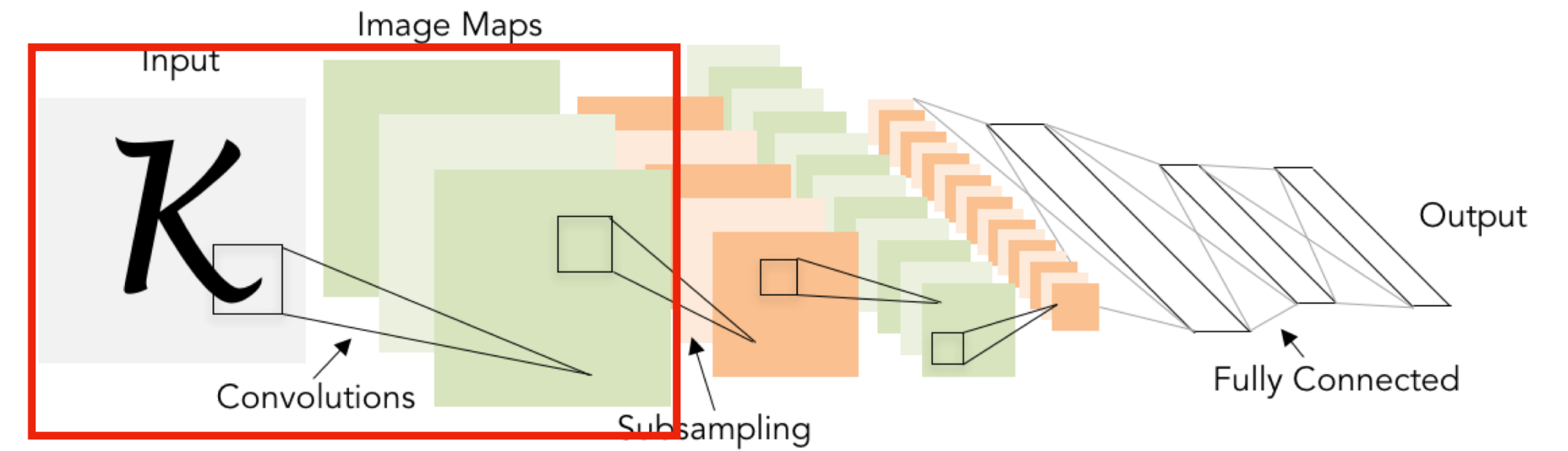






# Example: LeNet-5

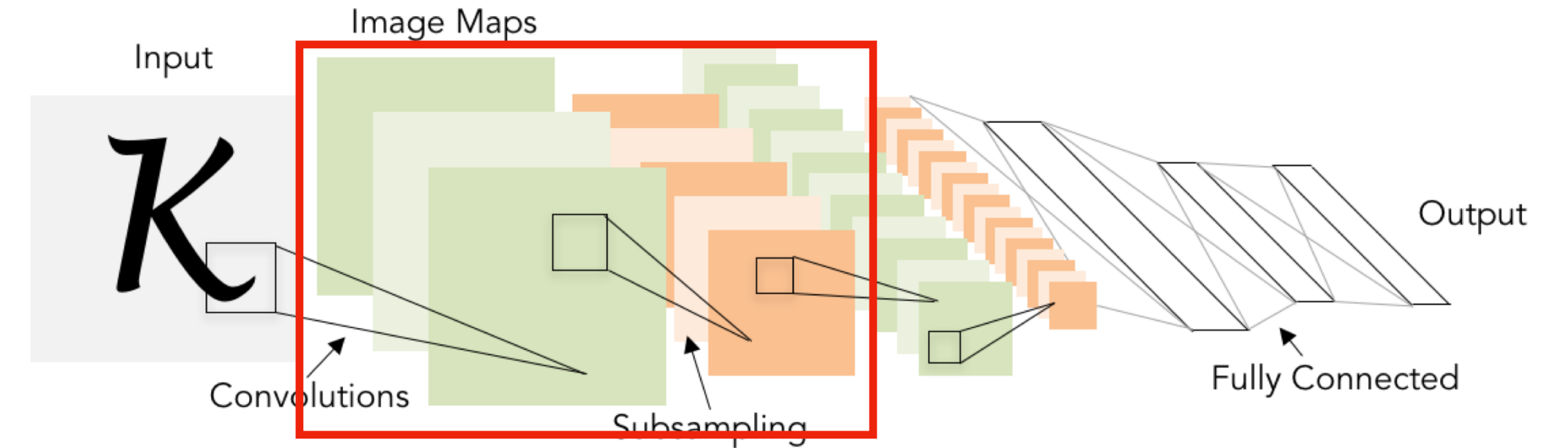
Layer	Output Size	Weight Size
Input	1 x 28 x 28	
Conv ( $C_{out}=20$ , $K=5$ , $P=2$ , $S=1$ )	20 x 28 x 28	20 x 1 x 5 x 5
ReLU	20 x 28 x 28	





# Example: LeNet-5

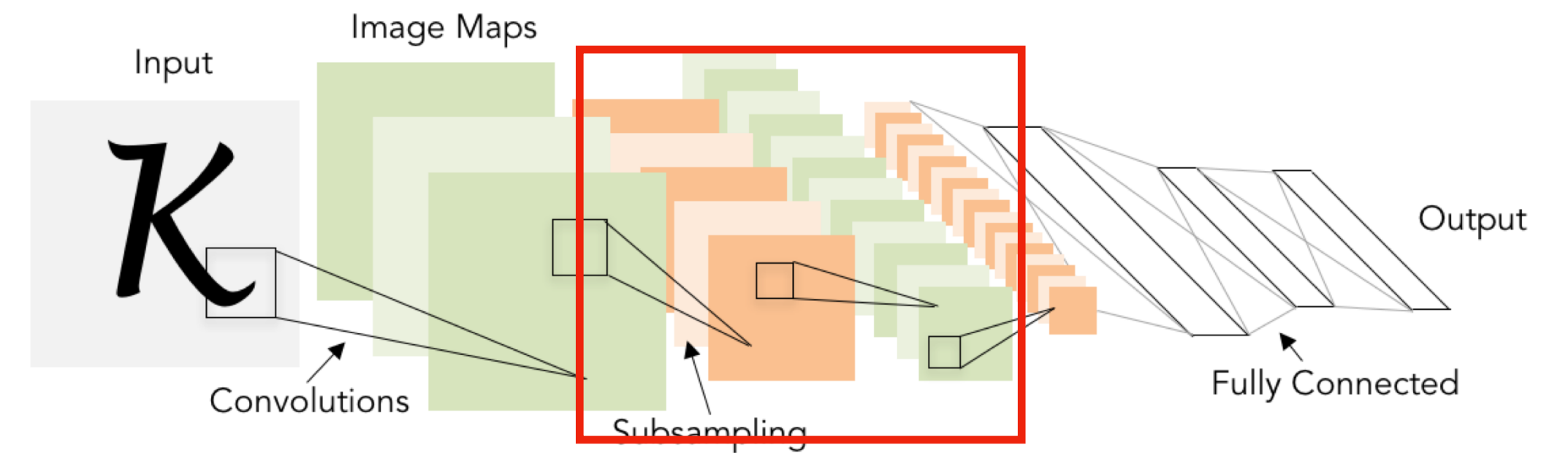
Layer	Output Size	Weight Size
Input	1 x 28 x 28	
Conv ( $C_{out}=20, K=5, P=2, S=1$ )	20 x 28 x 28	20 x 1 x 5 x 5
ReLU	20 x 28 x 28	
MaxPool( $K=2, S=2$ )	20 x 14 x 14	





# Example: LeNet-5

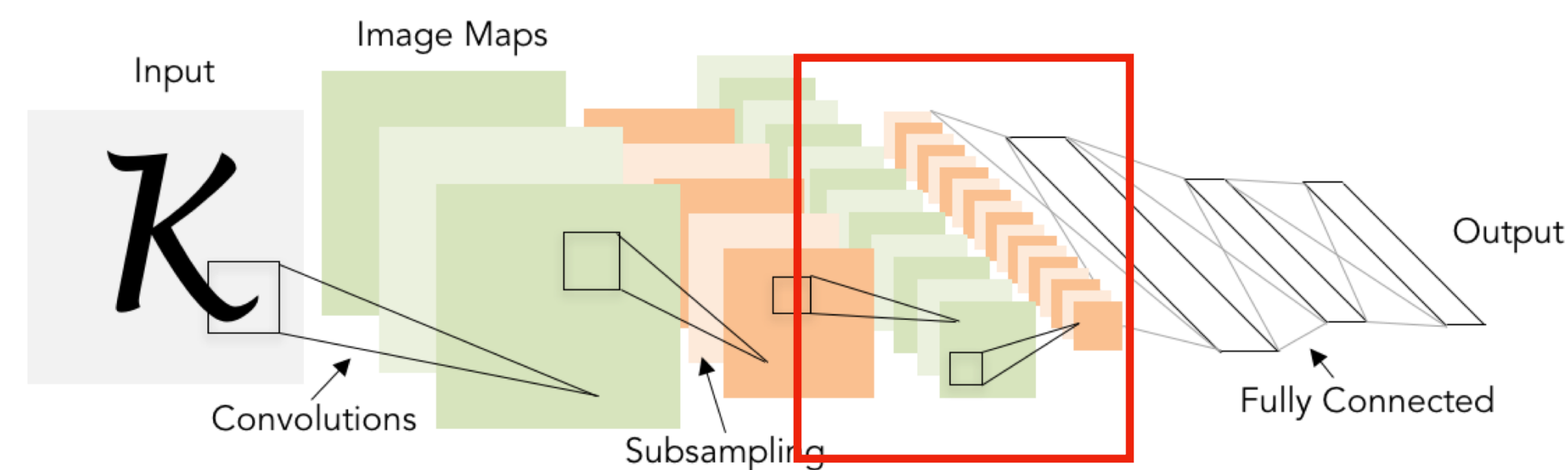
Layer	Output Size	Weight Size
Input	1 x 28 x 28	
Conv ( $C_{out}=20$ , $K=5$ , $P=2$ , $S=1$ )	20 x 28 x 28	20 x 1 x 5 x 5
ReLU	20 x 28 x 28	
MaxPool( $K=2$ , $S=2$ )	20 x 14 x 14	
Conv ( $C_{out}=50$ , $K=5$ , $P=2$ , $S=1$ )	50 x 14 x 14	50 x 20 x 5 x 5
ReLU	50 x 14 x 14	





# Example: LeNet-5

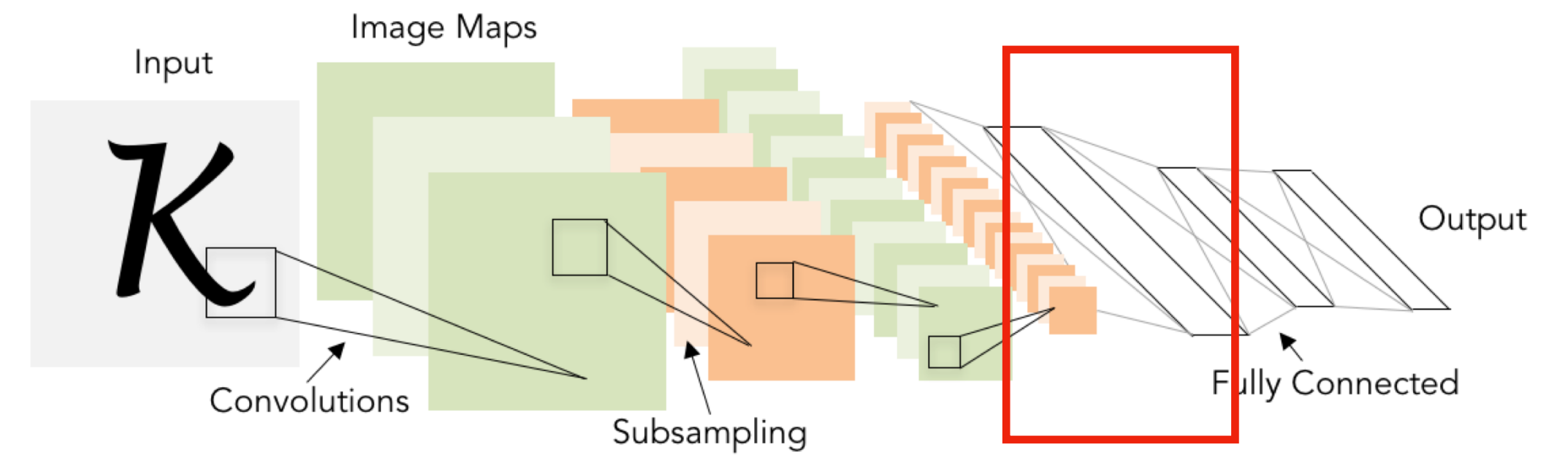
Layer	Output Size	Weight Size
Input	1 x 28 x 28	
Conv ( $C_{out}=20$ , $K=5$ , $P=2$ , $S=1$ )	20 x 28 x 28	20 x 1 x 5 x 5
ReLU	20 x 28 x 28	
MaxPool( $K=2$ , $S=2$ )	20 x 14 x 14	
Conv ( $C_{out}=50$ , $K=5$ , $P=2$ , $S=1$ )	50 x 14 x 14	50 x 20 x 5 x 5
ReLU	50 x 14 x 14	
MaxPool( $K=2$ , $S=2$ )	50 x 7 x 7	





# Example: LeNet-5

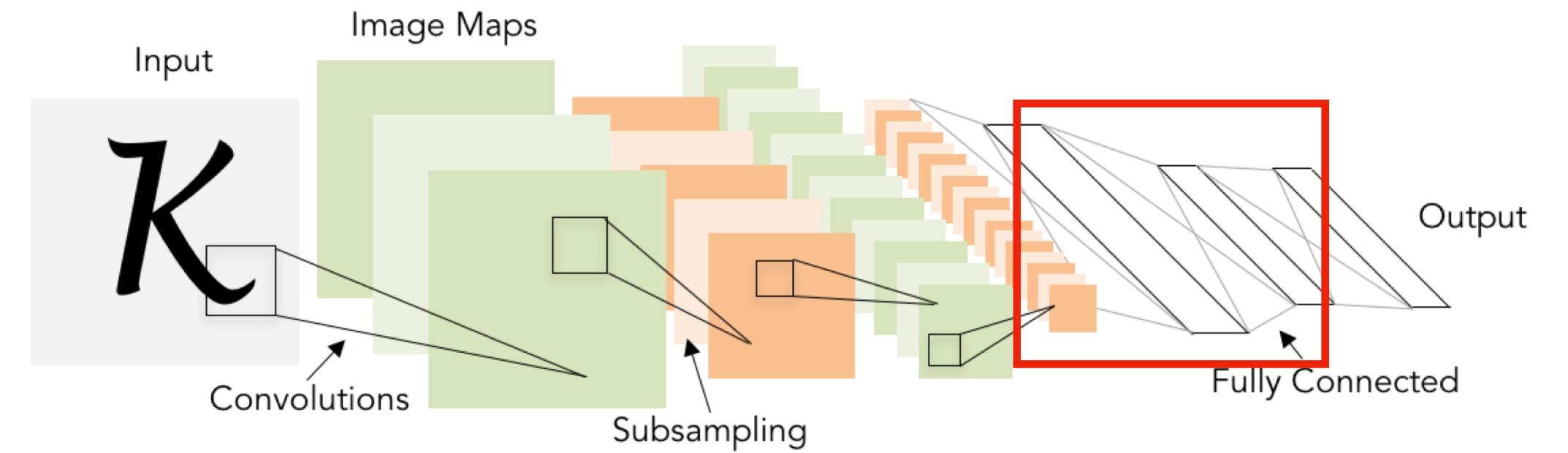
Layer	Output Size	Weight Size
Input	1 x 28 x 28	
Conv ( $C_{out}=20$ , $K=5$ , $P=2$ , $S=1$ )	20 x 28 x 28	20 x 1 x 5 x 5
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Conv ( $C_{out}=50$ , $K=5$ , $P=2$ , $S=1$ )	50 x 14 x 14	50 x 20 x 5 x 5
ReLU	50 x 14 x 14	
MaxPool( $K=2$ , $S=2$ )	50 x 7 x 7	
Flatten	2450	





# Example: LeNet-5

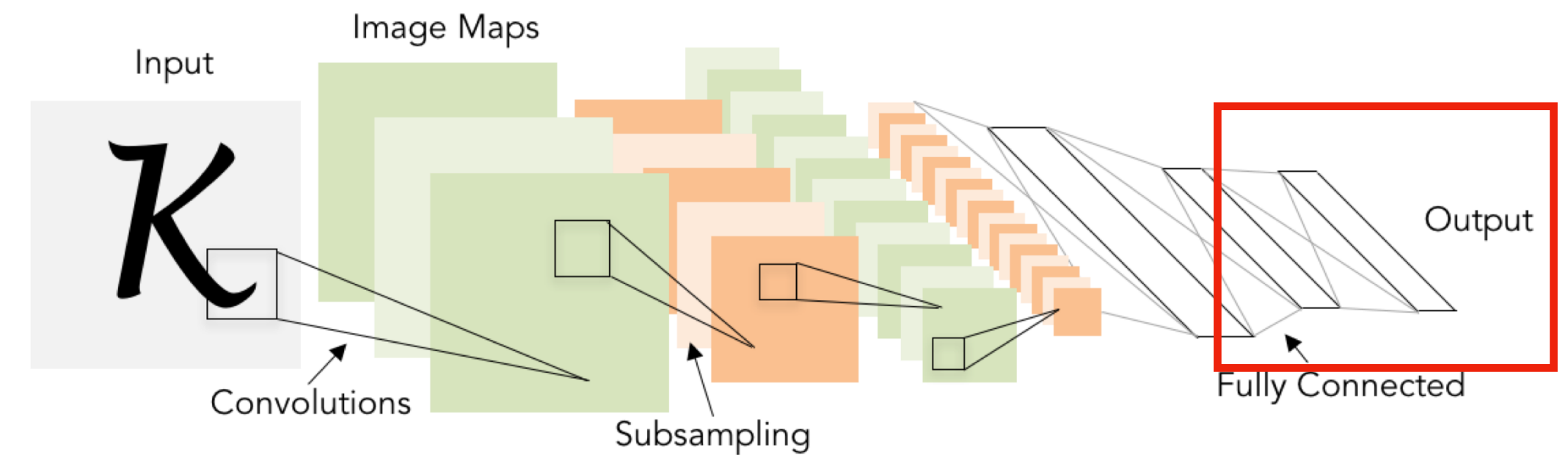
Layer	Output Size	Weight Size
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MaxPool( $K=2$ , $S=2$ )	50 x 7 x 7	
Flatten	2450	
Linear (2450 -> 500)	500	2450 x 500
ReLU	500	





# Example: LeNet-5

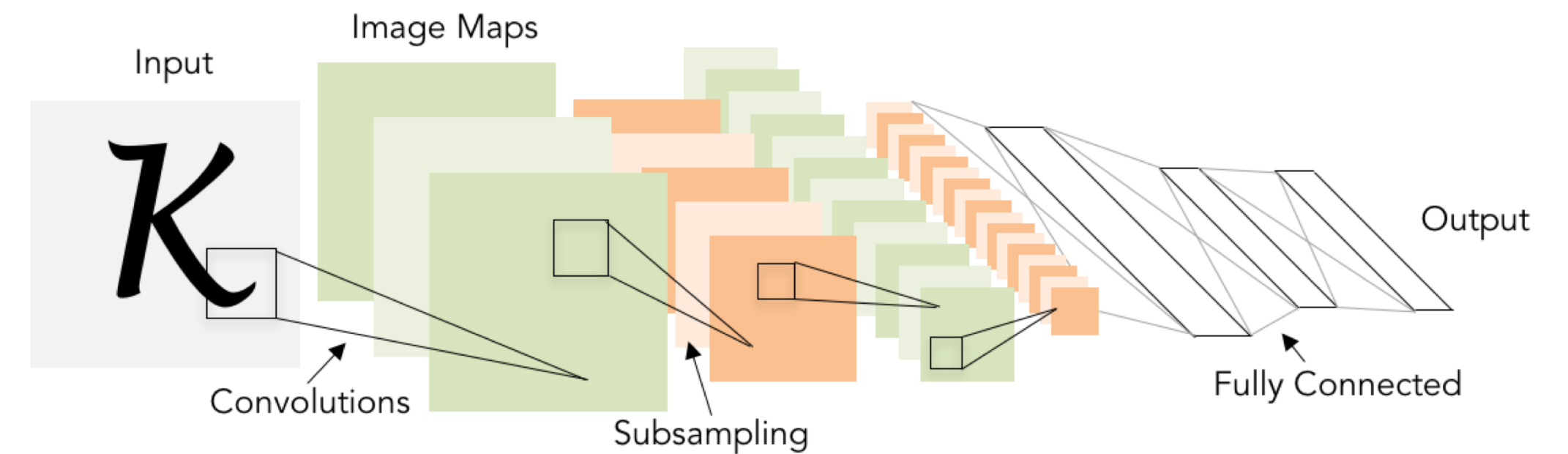
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ReLU	50 x 14 x 14	
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Flatten	2450	
Linear (2450 -> 500)	500	2450 x 500
ReLU	500	
Linear (500 -> 10)	10	500 x 10





# Example: LeNet-5

Layer	Output Size	Weight Size
Input	1 x 28 x 28	
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As we progress through the network:

Spatial size decreases

(using pooling or striped convolution)  
Number of channels increases

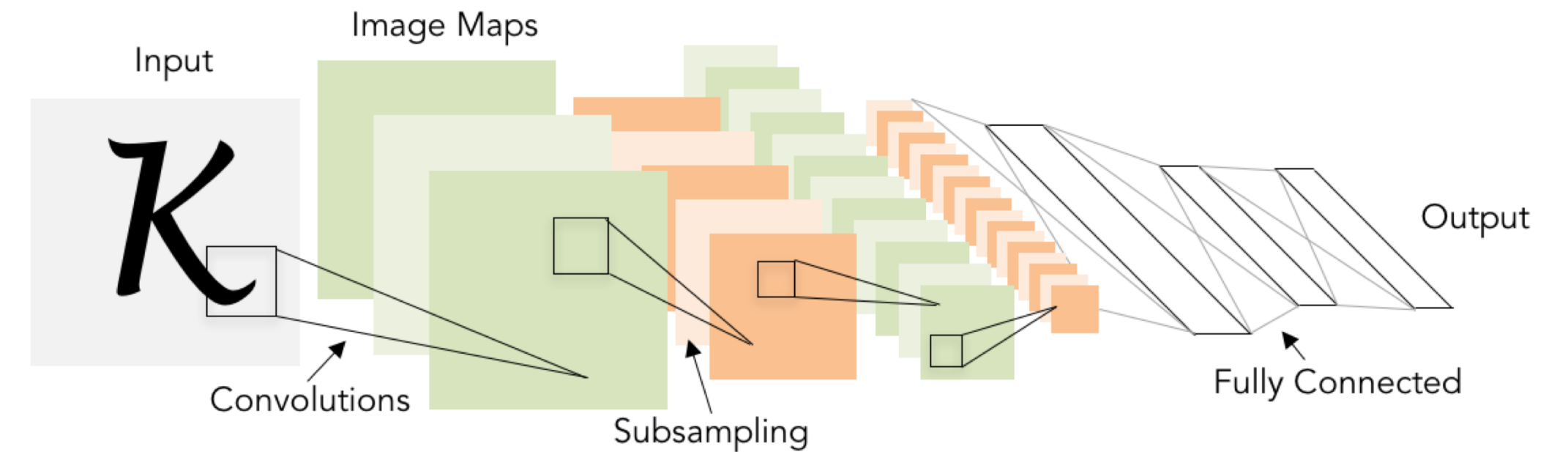
(total “volume” is preserved!)





# Example: LeNet-5

Layer	Output Size	Weight Size
Input	1 x 28 x 28	
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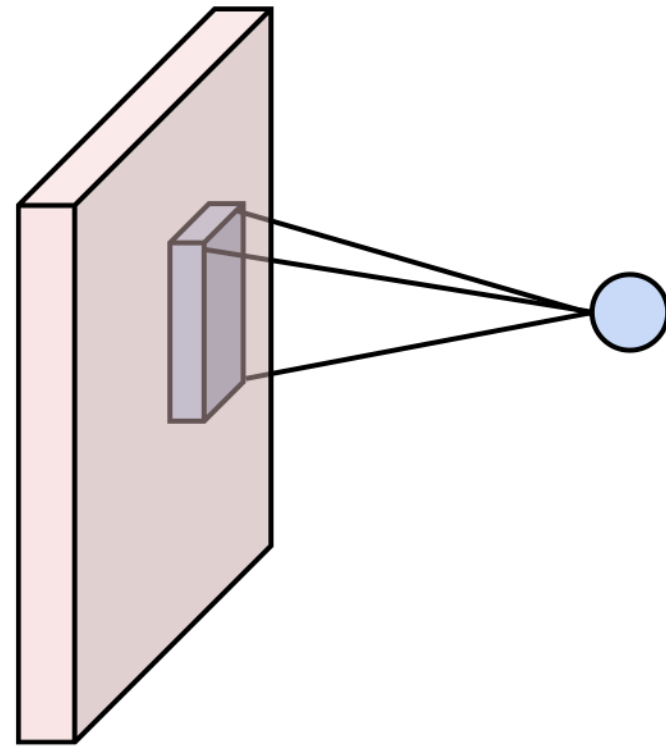
(total “volume” is preserved!)

Some modern architectures  
break this trend – stay tuned!

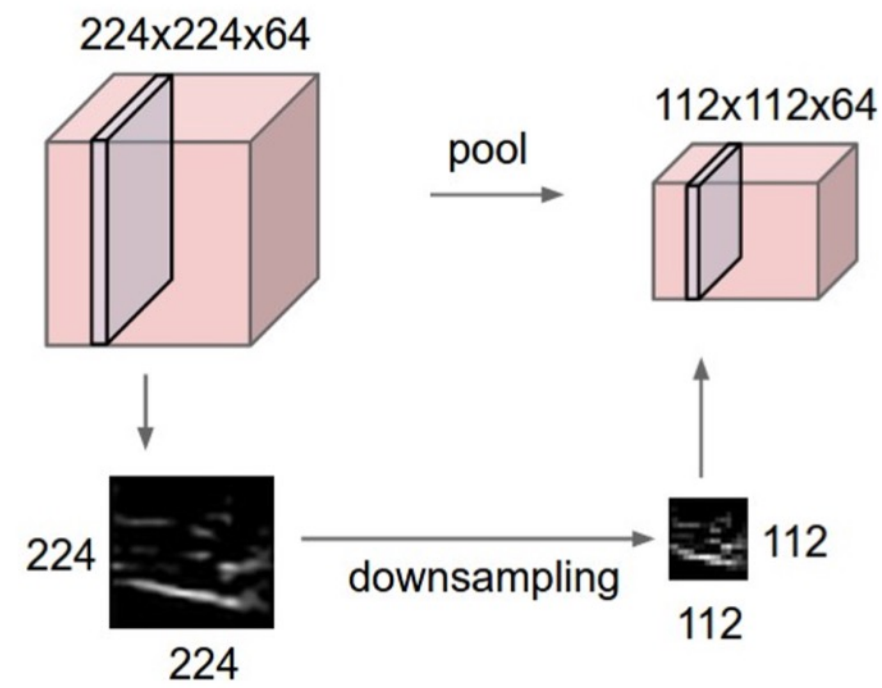


# Components of Convolutional Networks

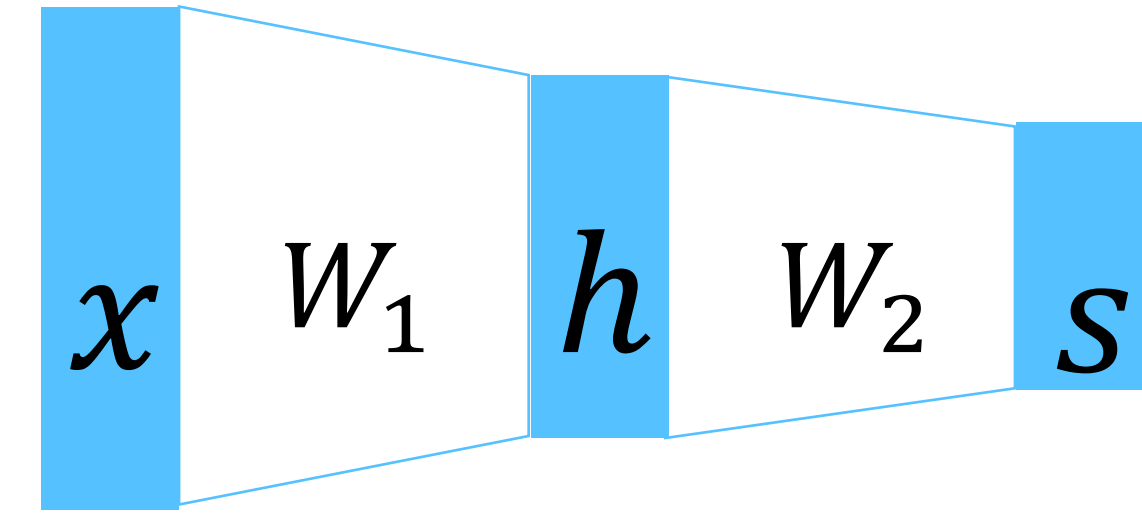
## Convolution Layers



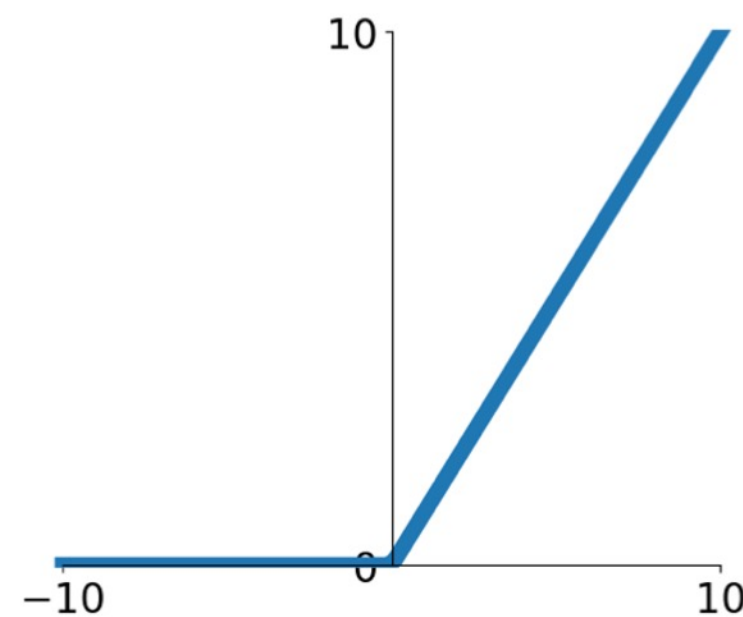
## Pooling Layers



## Fully-Connected Layers



## Activation Function



## Normalization

$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \epsilon}}$$

Problem:  
Deep  
Networks  
very hard to  
train



# Batch Normalization

---

Idea: “Normalize” the outputs of a layer so they have zero mean and unit variance

Why? Helps reduce “internal covariate shift”, improves optimization results

We can normalize a batch of activations using:

$$\hat{x} = \frac{x - E[x]}{\sqrt{Var[x]}}$$

Ioffe and Szegedy, “Batch normalization: Accelerating deep network training by reducing internal covariate shift”, ICML 2015



# Batch Normalization

---

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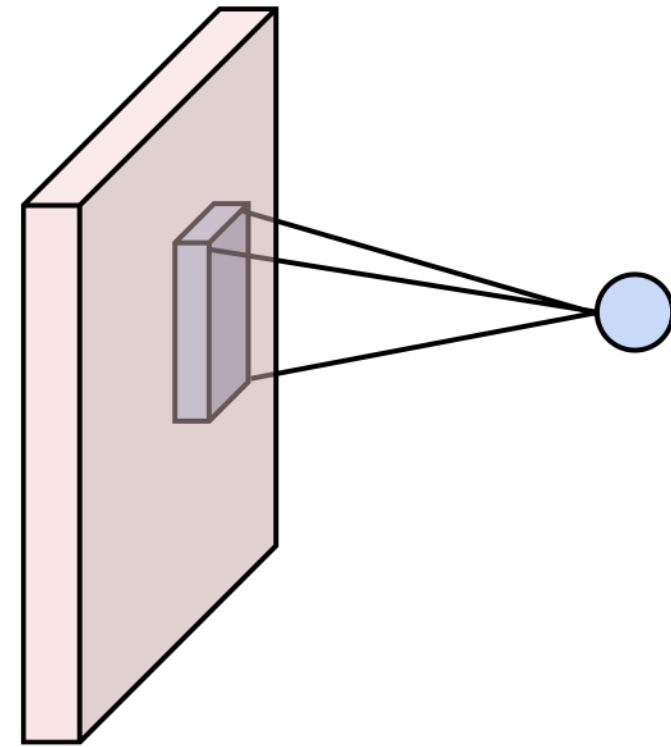
$$\hat{x} = \frac{x - E[x]}{\sqrt{Var[x]}}$$

This is a **differentiable function**, so we can use it as an operator in our networks and backprop through it!

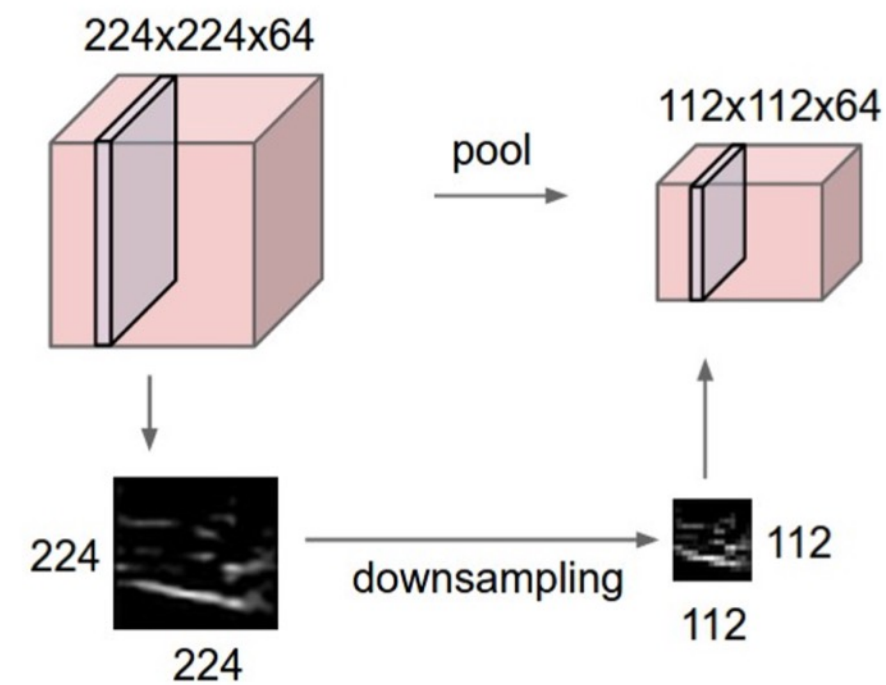


# Components of Convolutional Networks

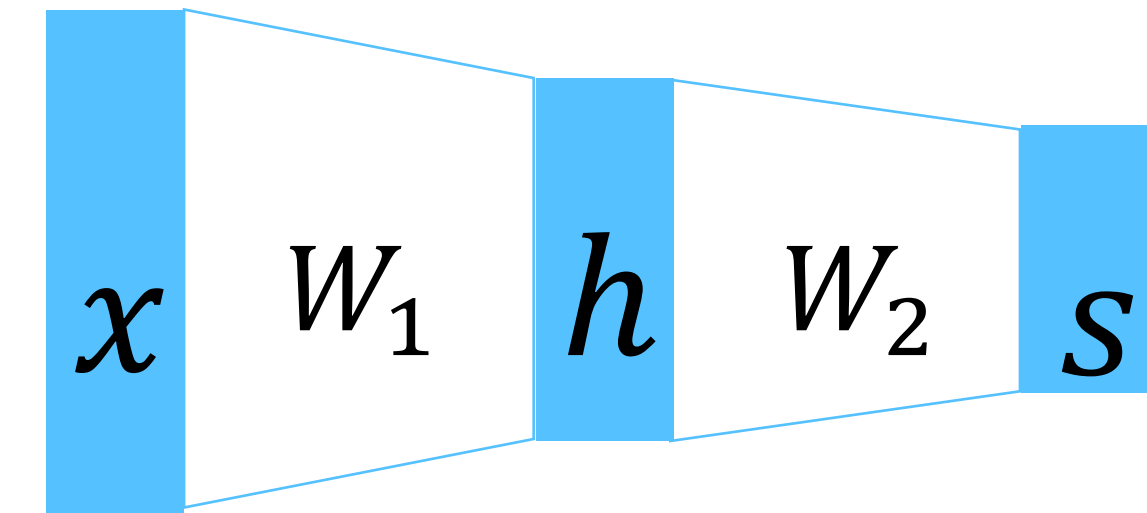
## Convolution Layers



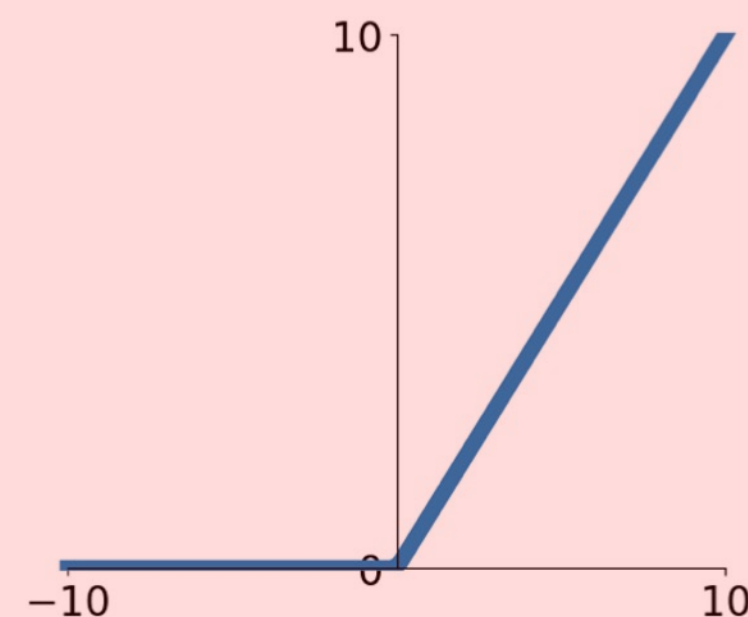
## Pooling Layers



## Fully-Connected Layers



## Activation Function



## Normalization

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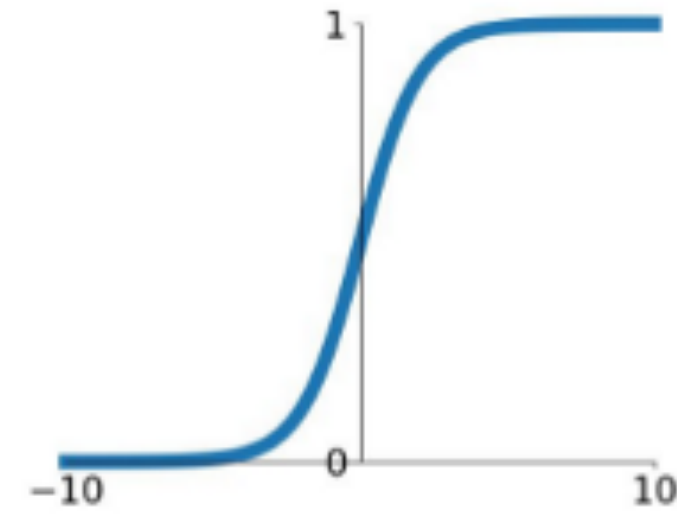
Problem:  
Deep  
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# Activation Functions

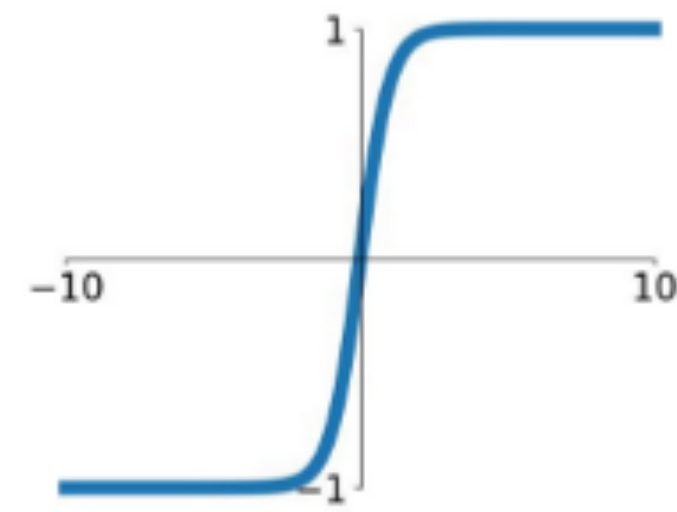
## Sigmoid

$$\sigma(x) = \frac{1}{1+e^{-x}}$$



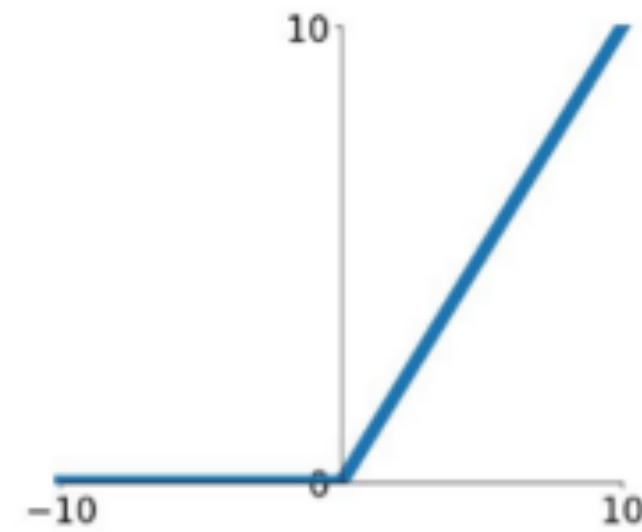
## tanh

$$\tanh(x)$$



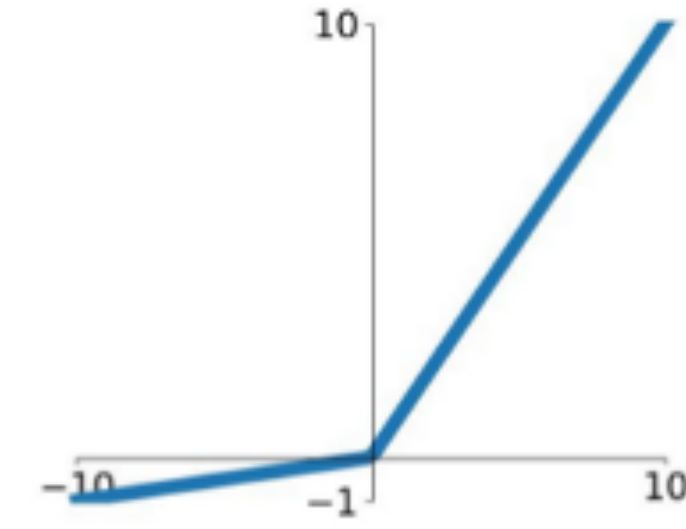
## ReLU

$$\max(0, x)$$



## Leaky ReLU

$$\max(0.1x, x)$$

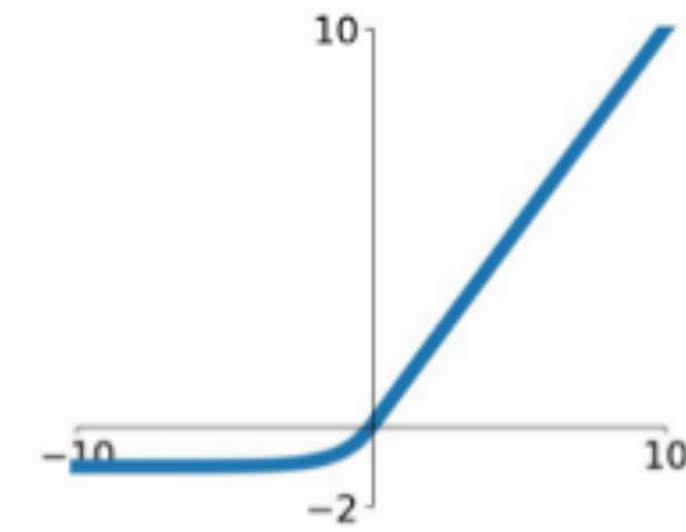


## Maxout

$$\max(w_1^T x + b_1, w_2^T x + b_2)$$

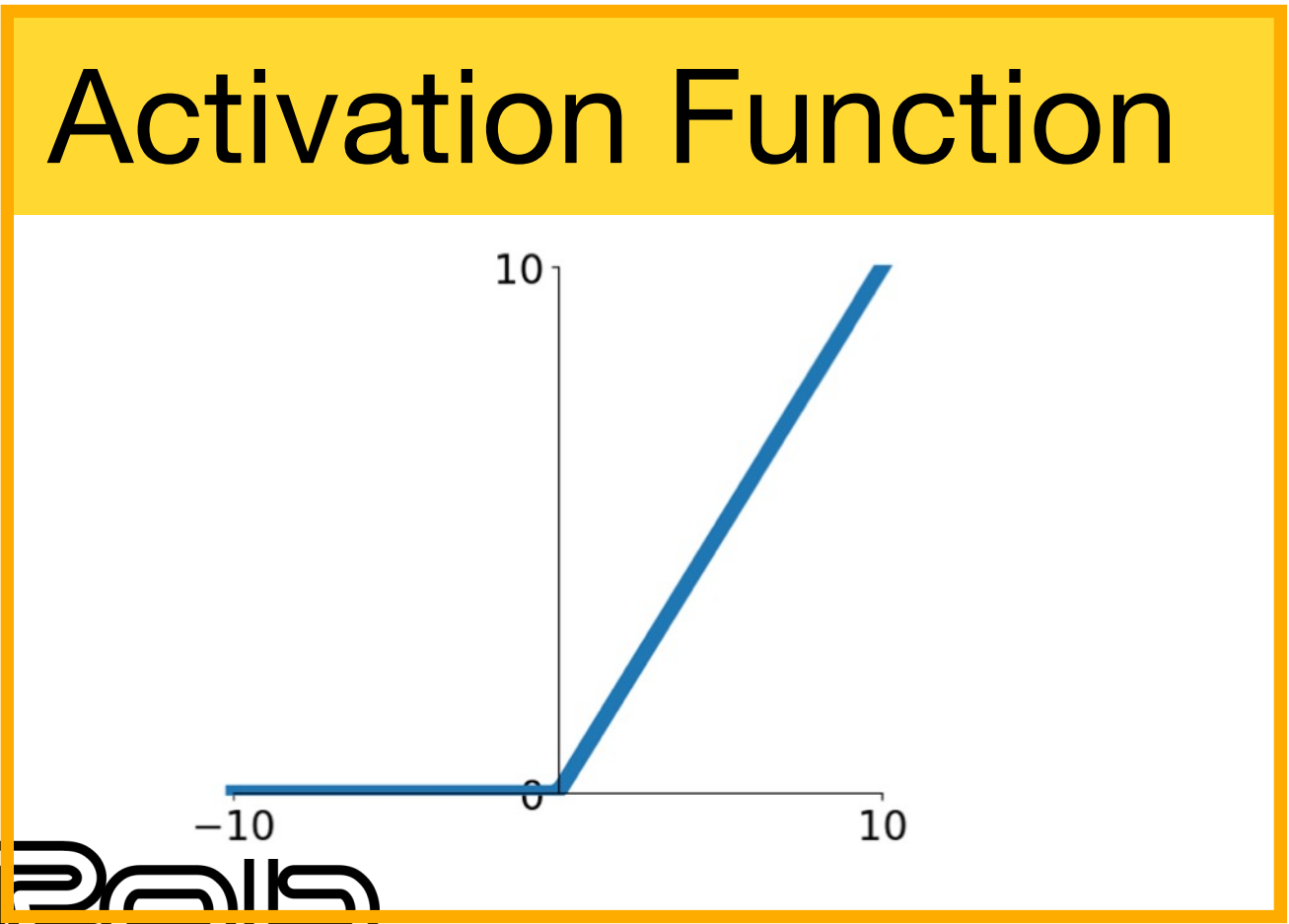
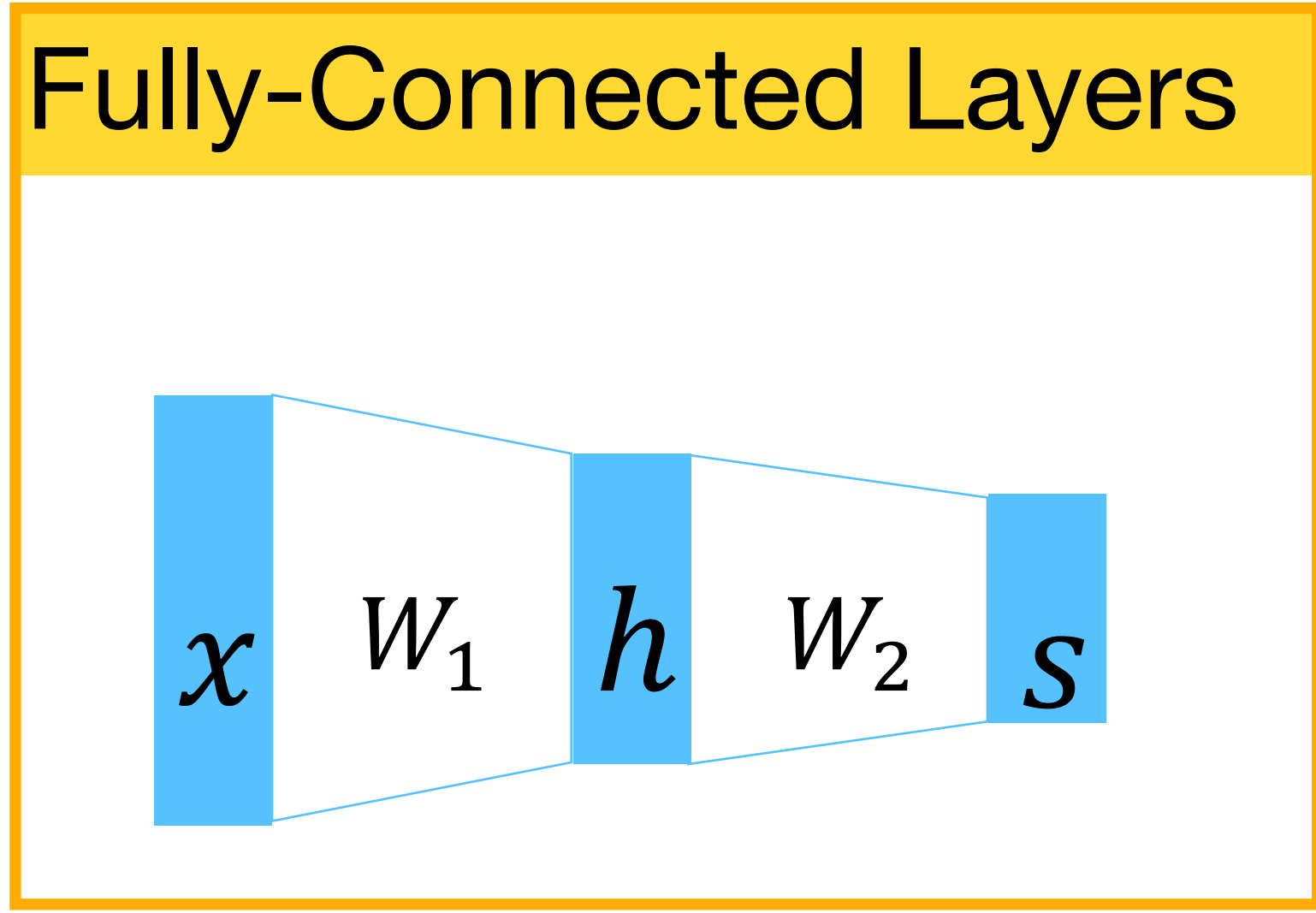
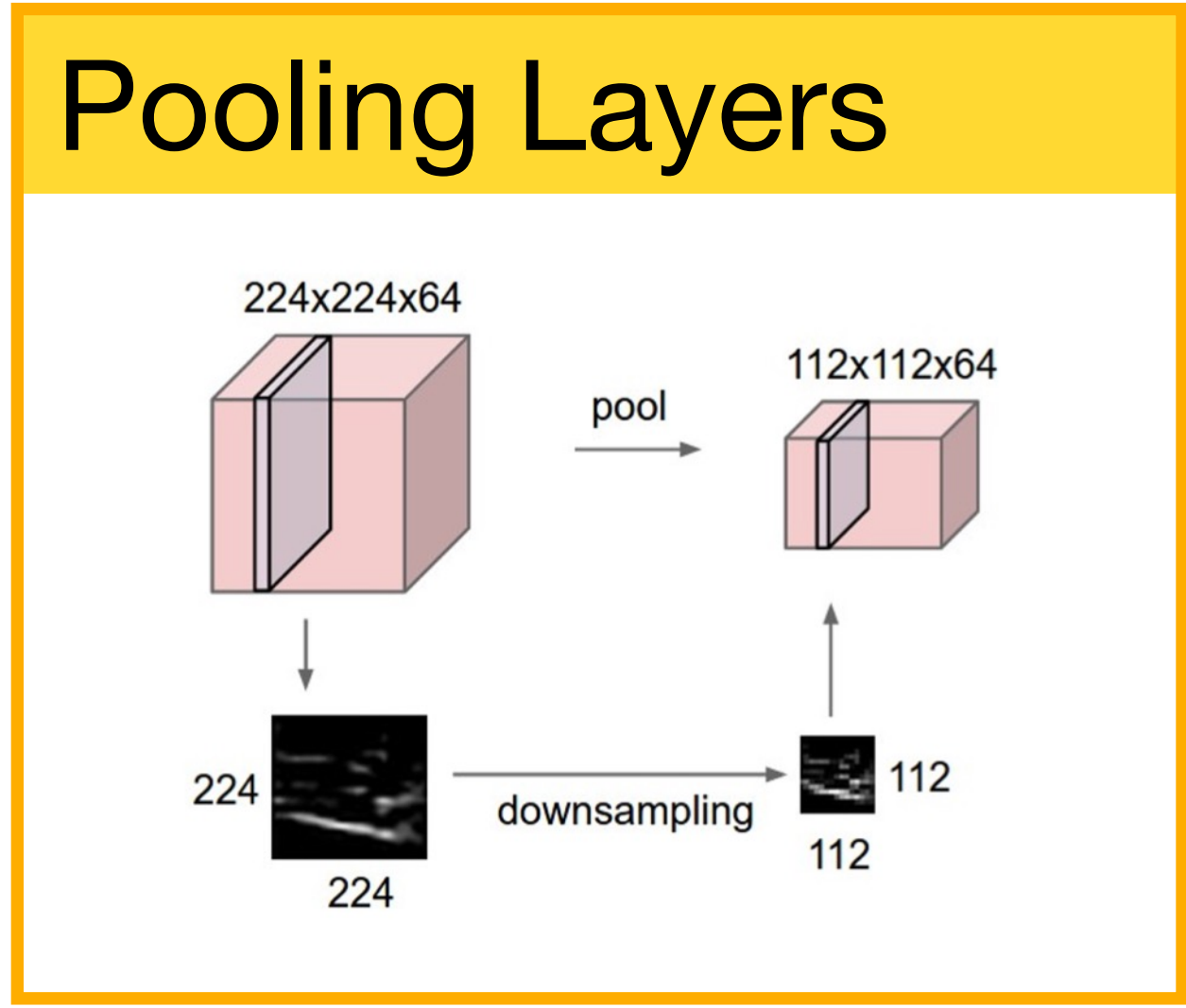
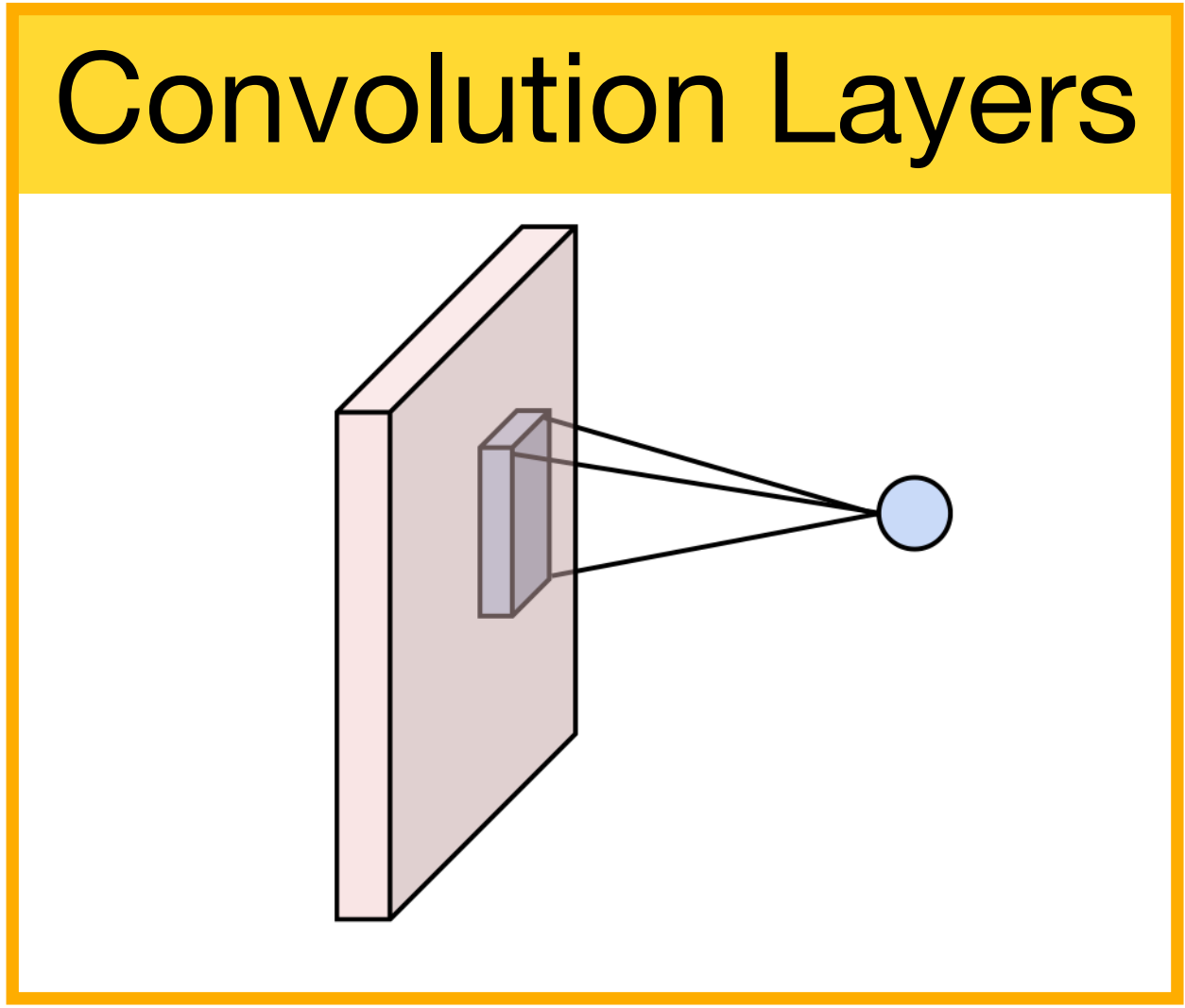
## ELU

$$\begin{cases} x & x \geq 0 \\ \alpha(e^x - 1) & x < 0 \end{cases}$$





# Summary: Components of Convolutional Networks



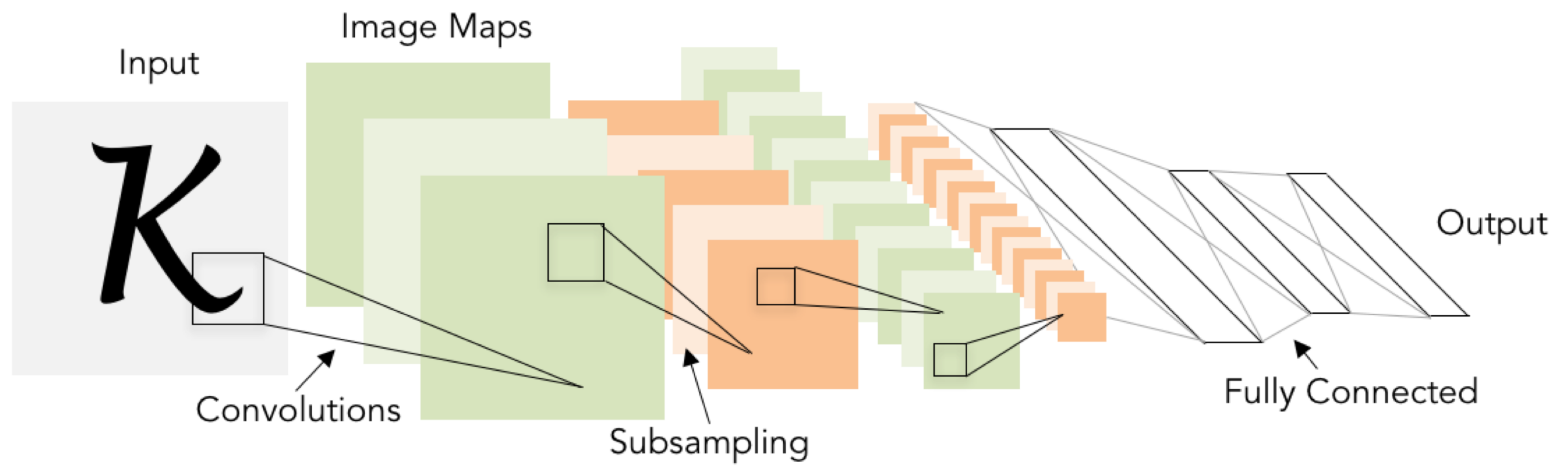
### Normalization

$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \epsilon}}$$



# Summary: Components of Convolutional Network

**Problem:** What is the right way to combine all these components?







# Project 1—Reminder

---

- Instructions and code available on the website
- Here: [deeprob.org/projects/project1/](https://deeprob.org/projects/project1/)
- Implement KNN, linear classifier, and fully connected NN
- **Due Thursday, Feb.1, 11:59 PM EST**
- **Discussion section: Your Thoughts?**
- **Late policy: 3 late tokens (24hrs each with no penalty); 25% deduction for every day the submission was late after using all three late tokens**



# Helpful References

---

- <https://cs231n.github.io/linear-classify/>
- <https://cs231n.github.io/optimization-1/>
- <https://cs231n.github.io/optimization-2/>
- [https://pytorch.org/tutorials/beginner/deep\\_learning\\_60min\\_bltz.html](https://pytorch.org/tutorials/beginner/deep_learning_60min_bltz.html)



# Final Project Overview

---

- Research-oriented final project
- Objectives
  - Gain experience reading literature
  - Reproduce published results
  - Propose a new idea and test the results!

completed in teams



# Final Project Deliverables

---

1. A written paper review
2. In-class paper presentation
3. Reproduce published results
4. Extend results with new idea, technique or dataset
5. Document results in written report



# (1) Paper Review and (2) Presentation

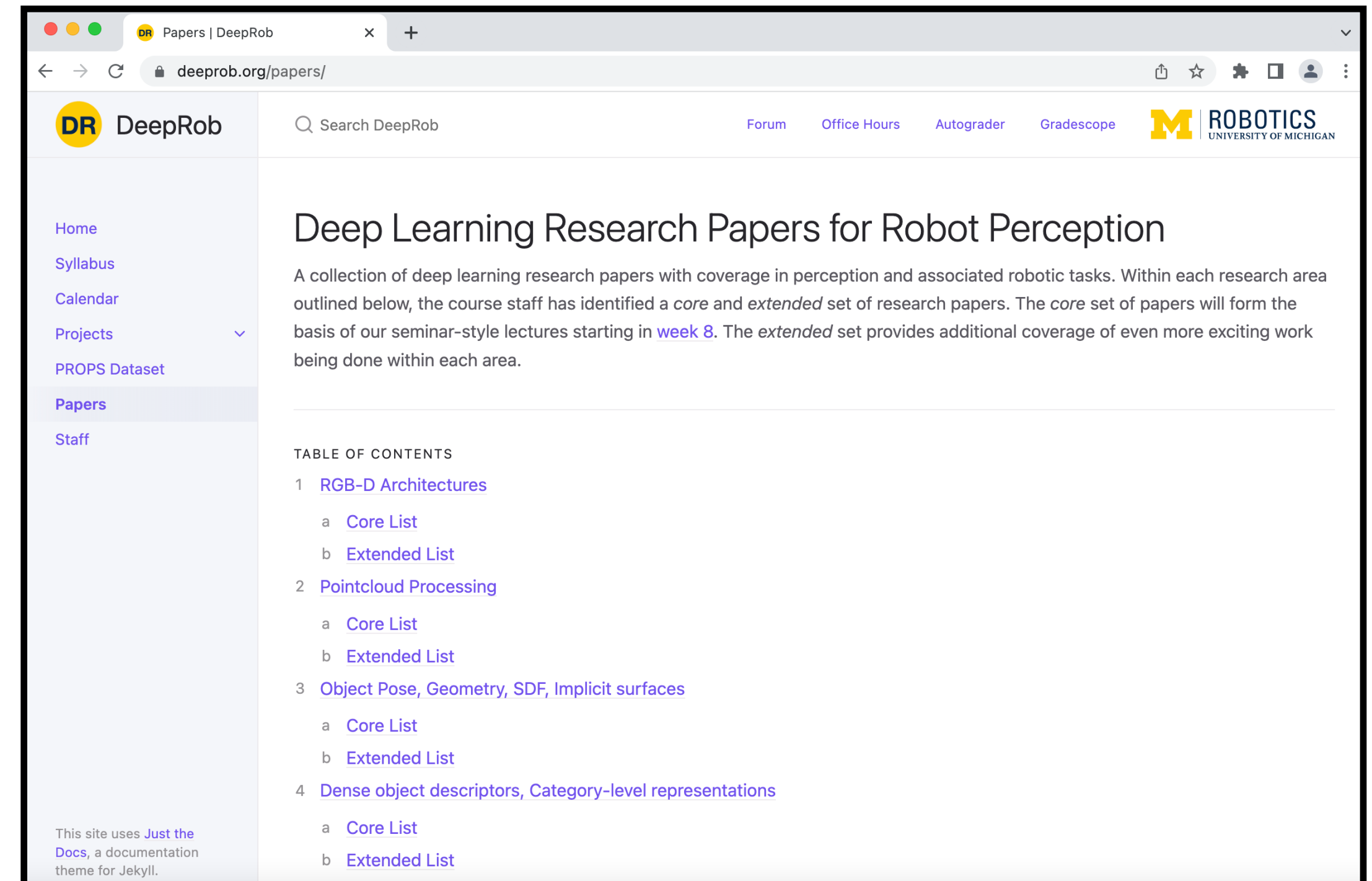
Final project teams will be based on overlapping interest

Students will choose from the 'core' list of papers on [course website](#)

Each team will be assigned one of the 'core' papers to review and present in-class

The 1-page paper review will be due **1-week before** the scheduled presentation

Presentation schedule will be based on paper topic as shown in [course calendar](#)



More details on review and presentation criteria in following lectures



# (3) Paper Reproduction and (4) Extension

---

Each team will choose a paper relating to deep learning and robot perception

Doesn't have to be same paper you presented in class

Then reimplement and reproduce at least one of the paper's published results (**not necessarily all the results**)

Then, each team will test one of their own ideas!

By extending the paper's model using new architecture or technique or dataset

**Your chance to experiment with deep learning and contribute to the field!**

More details on reproduction and extension in following lectures



## (5) Project Report

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- The final deliverable for your final project
- A report/paper
  - What problem within robot perception or manipulation?
  - What work has been done in this area?
  - What approach did you investigate?
  - What questions and directions exist for future work?



# DEEP ROB

Lecture 6  
Convolutional Neural Networks  
University of Michigan | Department of Robotics

